ChE 306: HEAT TRANSFER
FALL 2010
Homework #1 -SOLUTION
(80 points)
DUE: FRIDAY, AUGUST 27, 2010

Chapter 1 Problems

1. On a winter day, the inner surface of a glass window is 20 °C and the outer surface is 5 °C. The window is 7.5 mm thick, and measures 1 m by 3 m (length by width). Look up the thermal conductivity of plate glass in Appendix A3.

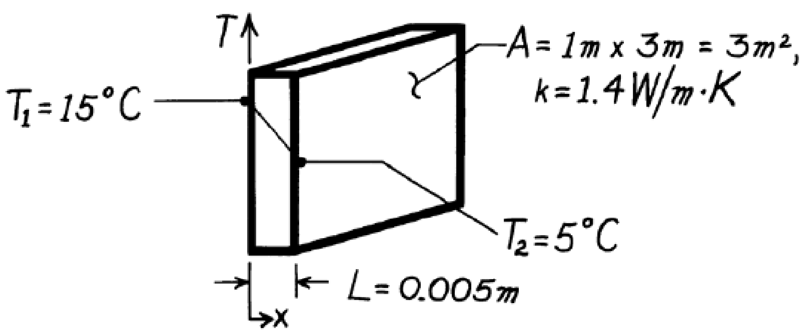
A. What is the rate of heat loss through the window (in Watts)?

The solution below illustrates how to solve, even though the numbers in this problem are different:

KNOWN: Inner and outer surface temperatures of a glass window of prescribed dimension:

FIND: Heat loss through window.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: Subject to the foregoing conditions the heat flux may be computed from Fourier's law, Eq. 1.2.

$$q''_x = k\frac{T_1 - T_2}{L}$$

$$q''_x = 1.4\frac{W}{m \cdot K}\frac{(15\text{-}5)^\circ C}{0.005m}$$

$$q''_x = 2800\ W/m^2.$$

Since the heat flux is uniform over the surface, the heat loss (rate) is

$$q = q''_x \times A$$

$$q = 2800\ W/m^2 \times 3m^2$$

$$q = 8400\ W. \quad <$$

COMMENTS: A linear temperature distribution exists in the glass for the prescribed conditions.

Here, T_1 is 20 °C, and the thickness, L, is 0.0075 m, so q = (3m)(1m)(1.4 W/m-K) (20-5 °C) / 0.0075 m
q = 8400 W

Note that no correction is needed for Kelvin/Celsius, since a temperature DIFFERENCE is used.

Note: This positive value of q is for a system where the x direction points from hot to cold temperatures.

B. Given the same thickness and dimensions of a slab of oak, with the same temperature conditions as the glass window, what is the heat loss (in Watts)? (Use Table A3 again) Compare your answer to (A) to see if your numbers make sense.

The only thing that changes is k. (k = 0.16 W/m-K)

$q = 0.16$ W/m-K $*3m^2 (20 - 5)/0.0075 = 960$ W

(or -960 W depending on how you defined the x direction).

Yes, wood is more insulating than glass, so heat loss is smaller.

C. Returning to the glass window, but on a summer day in Alabama, where the surface temperature on the outside surface of the glass is 104 ^oF and the inside surface temperature is 77 ^oF, what is the rate of heat transfer into the room through the glass (W)?

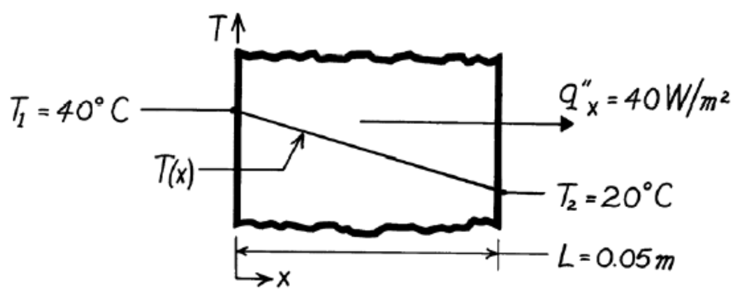
Converting Temperatures to Celsius, the driving force is 40 - 25 or 15 ^oC, so the dT is the same as in part (A), except that the direction is reversed. k and A are the same, so q = - 8400 W (or 8400 W going from outside to inside).

2. A rectangular slab of wood that is 50 mm thick has a known heat flux of 40 W/m^2. The surface temperatures on both sides of the wood slab are 40 and 20 ^oC, respectively.
A. What is the thermal conductivity of the wood?

KNOWN: Heat flux and surface temperatures associated with a wood slab of prescribed thickness.

FIND: Thermal conductivity, k, of the wood.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: Subject to the foregoing assumptions, the thermal conductivity may be determined from Fourier's law, Eq. 1.2. Rearranging,

$$k = q_x'' \frac{L}{T_1 - T_2} = 40 \frac{W}{m^2} \frac{0.05m}{(40\text{-}20)^{\circ} C}$$

$$k = 0.10 \ W/m \cdot K.$$ <

COMMENTS: Note that the °C or K temperature units may be used interchangeably when evaluating a temperature difference.

B. What thickness of an aluminum slab would be required to achieve the same heat flux for the same temperatures?

Given $q'' = 40 \ W/m^2$ and $T_1\text{-}T_2 = 20\ ^oC$ and looking up k(Al) = 237 W/m-K,

Using the equation above and solving for L:

*L = k * (T1-T2) / q" = 118.5 m. (MUCH, MUCH BIGGER!!!)*

3. Pressurized water at 50 ^oC flows inside a 5-cm inner diameter, 1 m long tube with the inside surface temperature maintained at 130 ^oC. The convective heat transfer coefficient between the water and the tube surface is 2000 W/m^2-^oC.

A. What is the convective heat transfer coefficient in Engineering (English) units (Btu/h-ft^2-^oF)?

Using the handy table in the back of your textbook (inside back cover), you will find that the conversion is 1 W/m2-K (which is the same as W/m^2-^oC) = 0.17611 Btu/h-ft^2-^oF,
so h = 2000 W/m^2-^oC = ***352.2 Btu/h-ft^2-^oF***

B. What is the heat transfer rate, q, from the tube to the water (in kW)?

*q = h A (Ts - Tinf) = 2000 W/m2-C * 3.14159 * 0.05 m * 1 m * (130 - 50 oC) * 1 kW/1000W*
q = 25.1 kW
*(surface area of a cylinder = pi * D * L... you'll need to know this for 306)*

C. Does the water exit the tube at 50 ^oC? If not, estimate the new exit temperature for a flow rate of 2.5 kg/s.

No... the water warms up... at least a little bit.
As an estimate we can equate q (from (B)) to the rate of change of internal energy (dU/dt)
dU/dt = mdot cp (Tout - Tin) (sensible energy change- there is no phase change). (mdot is mass flow rate)
you can look up heat capacity for water in table A.6 in our book. Use 50 ^oC (approx 325 K), and $c_{p,f}$ stands for heat capacity of liquid water ($c_{p,g}$ is for gas - phase water -or steam)
*So q = 25.1 kW = 2.5 kg/s * 4.182 kJ/kg-K * (Tout - 50 ^oC) (note: 1 kJ/s = 1kW)*
solving for ***T_{out} = 52.4 ^oC***
So it's not a bad approximation to determine q for water that stays at 50 ^oC, but it does warm some in the tube.

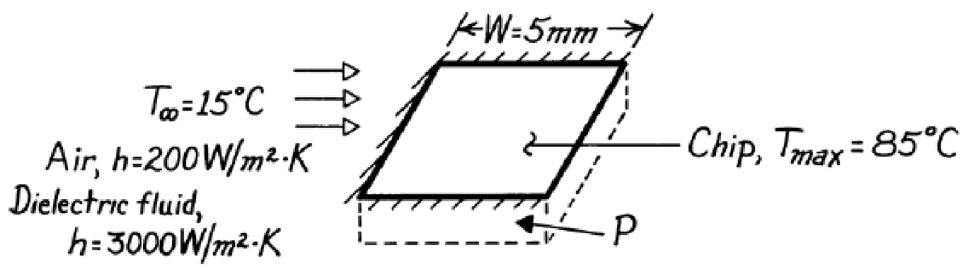
4. A computer chip must be cooled by the flow of air over its top surface. The chip is square and has a flat surface with 5 mm width. It is connected to the CPU by a plastic film so that it is well insulated on the bottom and the sides. The limiting (maximum) temperature to use the chip is 85 ^oC. Assume the air is at 15 ^oC and has a convection coefficient (h) of 50 W/m^2-K.
   A. What is the maximum rate of convective heat transfer (in W) from the chip?
   B. What is the maximum rate of convective heat transfer (in W) if the coolant is changed to a liquid with h = 300 W/m^2-K?
   C. If there were no convection, but radiation is considered as a way of cooling the chip, what is the maximum radiative cooling rate (in W) for the chip surface (which has an emissivity of 0.9)? You may assume that the temperature of the surroundings is 15^oC.
   D. Which situation would be better: combining air convection (part A) with radiative cooling (part C) or just using the liquid coolant (part B)?

A & B.

KNOWN: Chip width and maximum allowable temperature. Coolant conditions.

FIND: Maximum allowable chip power for air and liquid coolants.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat transfer from sides and bottom, (3) Chip is at a uniform temperature (isothermal), (4) Negligible heat transfer by radiation in air.

ANALYSIS: All of the electrical power dissipated in the chip is transferred by convection to the coolant. Hence,

$$P = q$$

and from Newton's law of cooling,

$$P = hA(T - T_\infty) = h\,W^2(T - T_\infty).$$

In *air*,

$$P_{max} = 200\ W/m^2\cdot K(0.005\ m)^2(85 - 15)\ °C = 0.35\ W. \qquad <$$

In the *dielectric liquid*

$$P_{max} = 3000\ W/m^2\cdot K(0.005\ m)^2(85-15)\ °C = 5.25\ W. \qquad <$$

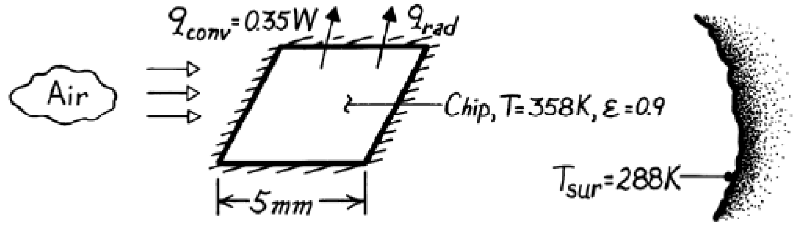
COMMENTS: Relative to liquids, air is a poor heat transfer fluid. Hence, in air the chip can dissipate far less energy than in the dielectric liquid.

C.

KNOWN: Chip width, temperature, and heat loss by convection in air. Chip emissivity and temperature of large surroundings.

FIND: Increase in chip power due to radiation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Radiation exchange between small surface and large enclosure.

ANALYSIS: Heat transfer from the chip due to net radiation exchange with the surroundings is

$$q_{rad} = \varepsilon W^2 \sigma \left(T^4 - T_{sur}^4\right)$$

$$q_{rad} = 0.9(0.005\ \text{m})^2\ 5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\left(358^4 - 288^4\right)\text{K}^4$$

$$q_{rad} = 0.0122\ \text{W}.$$

The percent increase in chip power is therefore

$$\frac{\Delta P}{P}\times100 = \frac{q_{rad}}{q_{conv}}\times100 = \frac{0.0122\ \text{W}}{0.350\ \text{W}}\times100 = 3.5\%. \qquad <$$

COMMENTS: For the prescribed conditions, radiation effects are small. Relative to convection, the effect of radiation would increase with increasing chip temperature and decreasing convection coefficient.

D. The heat loss (cooling) by air convection (found in part A) is 0.35 W. The cooling due to radiation is small, only 0.0122 W, so using A & C results in a net cooling rate of 0.3622 W. The liquid coolant (part B) is best!

Radiation is only really important when the source temperature is very hot (like the sun)!

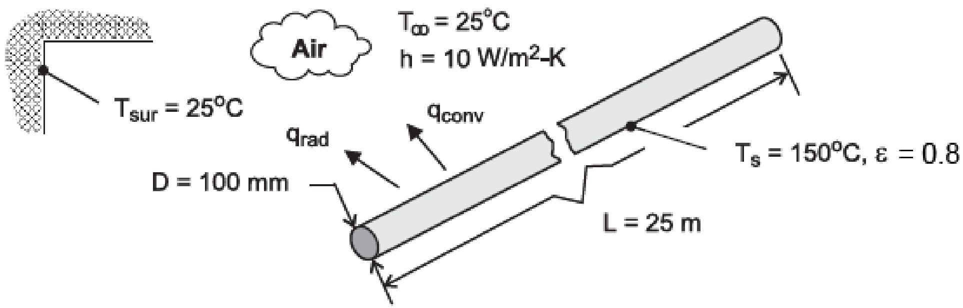
5. A steam pipe at an oil refinery passes through the air-conditioned control room, which has an air temperature of 25 °C and walls also at 25 °C. The pipe is uninsulated (metal is exposed to air) and the surface of the pipe is 150 °C. The pipe is 25 m long and has an outside diameter of approximately 4 inches (10 cm). Given that the convection coefficient for natural convection from the steam pipe to the room air is h = 10 W/m^2-K, and that the surface emissivity, ε, of 0.8:

A. Determine the rate of heat loss (q) from the pipe due to convection.
B. Determine the rate of heat loss (q) from the pipe due to radiation.
C. What is the total rate of heat loss from the pipe?
D. What would your answers to (A) and (B) be if the pipe were made of a different material with emissivity of 0.7?

KNOWN: Length, diameter, surface temperature and emissivity of steam line. Temperature and convection coefficient associated with ambient air. Efficiency and fuel cost for gas fired furnace.

FIND: (a) Rate of heat loss, (b) Annual cost of heat loss.

SCHEMATIC:



ASSUMPTIONS: (1) Steam line operates continuously throughout year, (2) Net radiation transfer is between small surface (steam line) and large enclosure (plant walls).

ANALYSIS: (a) From Eqs. (1.3a) and (1.7), the heat loss is

$$q = q_{conv} + q_{rad} = A\left[h\left(T_s - T_\infty\right) + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right]$$

where $A = \pi DL = \pi(0.1\text{m} \times 25\text{m}) = 7.85\text{m}^2$.

Hence,

$$q = 7.85\text{m}^2\left[10\ \text{W/m}^2\cdot\text{K}(150-25)\text{K} + 0.8\times 5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\left(423^4 - 298^4\right)\text{K}^4\right]$$

$$q = 7.85\text{m}^2\left(1,250 + 1,095\right)\text{W/m}^2 = \left(9813 + 8592\right)\text{W} = 18,405\ \text{W}$$ <

From above (a): q_{conv} = 9813 W,(b) q_{rad} = 8592 W, (c) 18,405 W

(d) If the emissivity changes, the heat loss to convection would not change (still 9813 W)
qrad = 0.7 * 5.67 * 10^{-8} * (423^4 - 298^4) = 7518 W.

Chapter 2 Problems

6. Work Problem 2.6 in Incropera & DeWitt.

KNOWN: Temperature dependence of the thermal conductivity, k(T), for heat transfer through a plane wall.

FIND: Effect of k(T) on temperature distribution, T(x).

ASSUMPTIONS: (1) One-dimensional conduction, (2) Steady-state conditions, (3) No internal heat generation.

ANALYSIS: From Fourier's law and the form of k(T),

$$q''_x = -k\frac{dT}{dx} = -(k_o + aT)\frac{dT}{dx}. \quad (1)$$

The shape of the temperature distribution may be inferred from knowledge of $d^2T/dx^2 = d(dT/dx)/dx$. Since q''_x is independent of x for the prescribed conditions,

$$\frac{dq''_x}{dx} = -\frac{d}{dx}\left[(k_o + aT)\frac{dT}{dx}\right] = 0$$

$$-(k_o + aT)\frac{d^2T}{dx^2} - a\left[\frac{dT}{dx}\right]^2 = 0.$$

Hence,

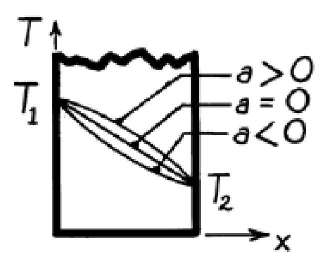
$$\frac{d^2T}{dx^2} = \frac{-a}{k_o + aT}\left[\frac{dT}{dx}\right]^2 \qquad \text{where } \begin{cases} k_o + aT = k > 0 \\ \left[\frac{dT}{dx}\right]^2 > 0 \end{cases}$$

from which it follows that for

$a > 0$: $d^2T/dx^2 < 0$

$a = 0$: $d^2T/dx^2 = 0$

$a < 0$: $d^2T/dx^2 > 0$.



COMMENTS: The shape of the distribution could also be inferred from Eq. (1). Since T decreases with increasing x,

$a > 0$: k decreases with increasing x => | dT/dx | increases with increasing x

$a = 0$: $k = k_o$ => dT/dx is constant

$a < 0$: k increases with increasing x => | dT/dx | decreases with increasing x.

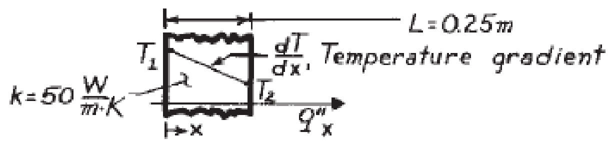
7. Work Problem 2.8 in Incropera & DeWitt.

PROBLEM 2.8

KNOWN: One-dimensional system with prescribed thermal conductivity and thickness.

FIND: Unknowns for various temperature conditions and sketch distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) No internal heat generation, (4) Constant properties.

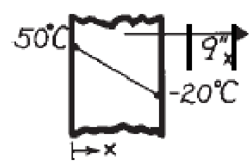
ANALYSIS: The rate equation and temperature gradient for this system are

$$q''_x = -k\frac{dT}{dx} \qquad \text{and} \qquad \frac{dT}{dx} = \frac{T_2 - T_1}{L}. \qquad (1,2)$$

Using Eqs. (1) and (2), the unknown quantities for each case can be determined.

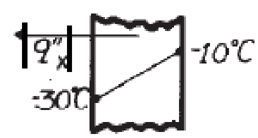
(a) $$\frac{dT}{dx} = \frac{(-20-50)\,K}{0.25m} = -280\ K/m$$

$$q''_x = -50\frac{W}{m\cdot K}\times\left[-280\frac{K}{m}\right] = 14.0\ kW/m^2. \qquad <$$



(b) $$\frac{dT}{dx} = \frac{(-10-(-30))\,K}{0.25m} = 80\ K/m$$

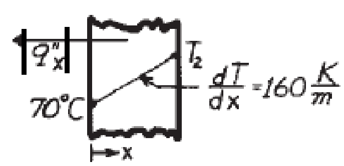
$$q''_x = -50\frac{W}{m\cdot K}\times\left[80\frac{K}{m}\right] = -4.0\ kW/m^2. \qquad <$$



(c) $$q''_x = -50\frac{W}{m\cdot K}\times\left[160\frac{K}{m}\right] = -8.0\ kW/m^2$$

$$T_2 = L\cdot\frac{dT}{dx} + T_1 = 0.25m\times\left[160\frac{K}{m}\right] + 70^\circ C.$$

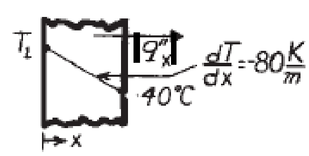
$$T_2 = 110^\circ C. \qquad <$$



(d) $$q''_x = -50\frac{W}{m\cdot K}\times\left[-80\frac{K}{m}\right] = 4.0\ kW/m^2$$

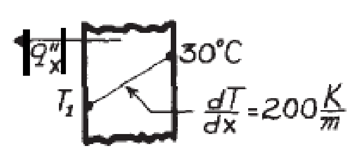
$$T_1 = T_2 - L\cdot\frac{dT}{dx} = 40^\circ C - 0.25m\left[-80\frac{K}{m}\right]$$

$$T_1 = 60^\circ C. \qquad <$$



(e) $$q''_x = -50\frac{W}{m\cdot K}\times\left[200\frac{K}{m}\right] = -10.0\ kW/m^2$$

$$T_1 = T_2 - L\cdot\frac{dT}{dx} = 30^\circ C - 0.25m\left[200\frac{K}{m}\right] = -20^\circ C. \qquad <$$



8. Work Problem 2.16 in Incropera & DeWitt.

Determine the required thermal conductivity of the manufacturer's insulation, and use Appendix A3 (p 937-940) to determine what material the insulation might be made of.

PROBLEM 2.16

KNOWN: Different thicknesses of three materials: rock, 18 ft; wood, 15 in; and fiberglass insulation, 6 in.

FIND: The insulating quality of the materials as measured by the R-value.

PROPERTIES: *Table A-3* (300K):

Material	Thermal conductivity, W/m·K
Limestone	2.15
Softwood	0.12
Blanket (glass, fiber 10 kg/m^3)	0.048

ANALYSIS: The R-value, a quantity commonly used in the construction industry and building technology, is defined as

$$R \equiv \frac{L(\text{in})}{k\left(\text{Btu}\cdot\text{in/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}\right)}.$$

The R-value can be interpreted as the thermal resistance of a 1 ft^2 cross section of the material. Using the conversion factor for thermal conductivity between the SI and English systems, the R-values are:

Rock, Limestone, 18 ft:

$$R = \frac{18\ \text{ft} \times 12\dfrac{\text{in}}{\text{ft}}}{2.15\dfrac{\text{W}}{\text{m}\cdot\text{K}} \times 0.5778\dfrac{\text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}}{\text{W/m}\cdot\text{K}} \times 12\dfrac{\text{in}}{\text{ft}}} = 14.5\ \left(\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}\right)^{-1} \quad <$$

Wood, Softwood, 15 in:

$$R = \frac{15\ \text{in}}{0.12\dfrac{\text{W}}{\text{m}\cdot\text{K}} \times 0.5778\dfrac{\text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}}{\text{W/m}\cdot\text{K}} \times 12\dfrac{\text{in}}{\text{ft}}} = 18\ \left(\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}\right)^{-1} \quad <$$

Insulation, Blanket, 6 in:

$$R = \frac{6\ \text{in}}{0.048\dfrac{\text{W}}{\text{m}\cdot\text{K}} \times 0.5778\dfrac{\text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}}{\text{W/m}\cdot\text{K}} \times 12\dfrac{\text{in}}{\text{ft}}} = 18\ \left(\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}\right)^{-1} \quad <$$

COMMENTS: The R-value of 19 given in the advertisement is reasonable.

For an R value of 19 and a thickness of 6 inches,
$k = 6\ \text{in} / \{(19\text{in/Btu/ft}^o\text{F}) \times 0.5778 \times 12\ \text{in/ft}\} = 0.0455\ \text{W/m-K}$
this is similar to a blanket, or clay tile, or some plastering materials, or cork.