

# ChE 306: HEAT TRANSFER

FALL 2010

Homework #3: Chapters 4 & 5

(70 points)

DUE: MONDAY, SEPTEMBER 13

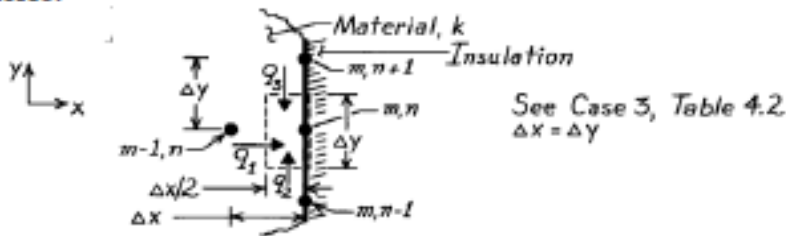
## 1. Work Incropera and DeWitt Problem 4.33

You may assume the material has a constant thermal conductivity,  $k$ .

**KNOWN:** Plane surface of two-dimensional system.

**FIND:** The finite-difference equation for nodal point on this boundary when (a) insulated; compare result with Eq. 4.42, and when (b) subjected to a constant heat flux.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Two-dimensional, steady-state conduction with no generation, (2) Constant properties, (3) Boundary is adiabatic.

**ANALYSIS:** (a) Performing an energy balance on the control volume,  $(\Delta x/2) \cdot \Delta y$ , and using the conduction rate equation, it follows that

$$\dot{E}_{in} - \dot{E}_{out} = 0 \quad q_1 + q_2 + q_3 = 0 \quad (1,2)$$

$$k(\Delta y) \frac{T_{m-1,n} - T_{m,n}}{\Delta x} + k \left[ \frac{\Delta x}{2} \cdot 1 \right] \frac{T_{m,n-1} - T_{m,n}}{\Delta y} + k \left[ \frac{\Delta x}{2} \cdot 1 \right] \frac{T_{m,n+1} - T_{m,n}}{\Delta y} = 0 \quad (3)$$

Note that there is no heat rate across the control volume surface at the insulated boundary.

Recognizing that  $\Delta x = \Delta y$ , the above expression reduces to the form

$$2T_{m-1,n} + T_{m,n-1} + T_{m,n+1} - 4T_{m,n} = 0 \quad (4) <$$

The Eq. 4.42 of Table 4.2 considers the same configuration but with the boundary subjected to a convection process. That is,

$$(2T_{m-1,n} + T_{m,n-1} + T_{m,n+1}) + \frac{2h\Delta x}{k} T_{\infty} - 2 \left[ \frac{h\Delta x}{k} + 2 \right] T_{m,n} = 0 \quad (5)$$

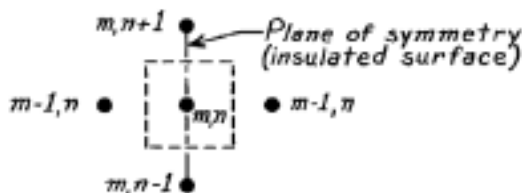
Note that, if the boundary is insulated,  $h = 0$  and Eq. 4.42 reduces to Eq. (4).

(b) If the surface is exposed to a constant heat flux,  $q_0''$ , the energy balance has the form

$q_1 + q_2 + q_3 + q_0'' \cdot \Delta y = 0$  and the finite difference equation becomes

$$2T_{m-1,n} + T_{m,n-1} + T_{m,n+1} - 4T_{m,n} = -\frac{2q_0'' \Delta x}{k} \quad <$$

**COMMENTS:** Equation (4) can be obtained by using the "interior node" finite-difference equation, Eq. 4.29, where the insulated boundary is treated as a symmetry plane as shown below.



2. A portion of a solid material is shown below. The material is a bar of gold that is fully insulated on its right side. The top and bottom of the section shown extend beyond the nodes shown in the picture, and the boundary on the left side of the gold is air at a temperature of 75 °F. You may assume there is no heat generation or consumption within the gold bar.

Nine nodes are shown on a square grid with  $\Delta x = \Delta y = 0.025$  m

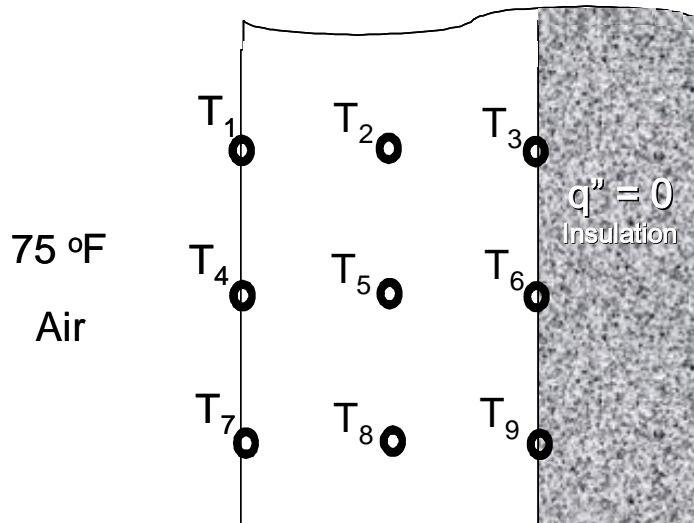
**Given the following data, determine temperatures  $T_5$  and  $T_6$ .**

Properties of Gold

density = 19300 kg/m<sup>3</sup>  
 heat capacity = 129 J/kg-K  
 thermal conductivity = 317 W/m-K  
 thermal diffusivity =  $1.27 \times 10^{-4}$  m<sup>2</sup>/s

Known Node Temperatures

$T_1 = 42.0$  K ;  $T_2 = 58.0$  K ;  
 $T_3 = 66.0$  K ;  $T_4 = 40.0$  K ,  
 $T_7 = 39.0$  K ;  $T_8 = 47.0$  K ;  $T_9 = 59.0$  K



Air Properties

convective heat transfer coefficient = 12.4 W/m<sup>2</sup>-K  
 density = 1.08 kg/m<sup>3</sup>  
 thermal conductivity = 0.0284 W/m-K

*Given  $T_1$ ,  $T_4$ ,  $T_7$  and convective conditions at the surface, a node balance around node 4 can be used to solve for  $T_5$ .*

*Using Eq. 4.42,  $(2 T_5 + T_1 + T_7) + 2 h \Delta x / k T_{Air} - 2 (h \Delta x / k + 2) T_4 = 0$*

*Everything is known in the equation except  $T_5$ .*

*Convert 75 °F to Kelvin: 297K*

$$(2 T_5 + 42 + 39) + 2 (12.4) (0.025) / (317) * 297 K - 2 (12.4 * 0.025) / 317 + 2) (40) = 0$$

$$T_5 = 39.2 K$$

*Now, to find  $T_6$ , can do a node balance around  $T_5$  (simplest method):*

$$T_5 = (T_2 + T_4 + T_6 + T_8) / 4$$

$$T_6 = 12 K$$

*Without my correction to remove  $T_3$  &  $T_9$ , the problem is overspecified: so there would have been another way to work it:*

*using  $T_3$  &  $T_9$ , and doing a node balance around  $T_6$  yields Equation 4.44 with  $q'' = 0$*

$$2 * T_5 + T_3 + T_9 - 4 * T_6 = 0$$

*$T_6 = 50.8$  K (quite a bit different than earlier... there were too many knowns given in the problem).*

### 3. Work Incropera and DeWitt Problem 4.48

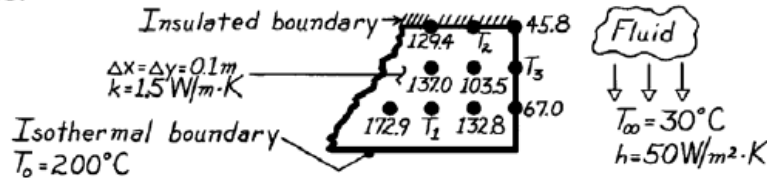
For part (b), recall that each node temperature represents the temperature for a unit square surrounding it in the solid; thus, the heat transfer rate will be the sum of convective heat loss from each surface segment of the grid.

#### PROBLEM 4.48

**KNOWN:** Steady-state temperatures ( $^{\circ}\text{C}$ ) associated with selected nodal points in a two-dimensional system.

**FIND:** (a) Temperatures at nodes 1, 2 and 3, (b) Heat transfer rate per unit thickness from the system surface to the fluid.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Steady-state, two-dimensional conduction, (2) Constant properties.

**ANALYSIS:** (a) Using the finite-difference equations for Nodes 1, 2 and 3:

Node 1, Interior node, Eq. 4.29:  $T_1 = \frac{1}{4} \cdot \sum T_{\text{neighbors}}$

$$T_1 = \frac{1}{4}(172.9 + 137.0 + 132.8 + 200.0)^{\circ}\text{C} = 160.7^{\circ}\text{C}$$

Node 2, Insulated boundary, Eq. 4.46 with  $h = 0$ ,  $T_{m,n} = T_2$

$$T_2 = \frac{1}{4}(T_{m-1,n} + T_{m+1,n} + 2T_{m,n-1})$$

$$T_2 = \frac{1}{4}(129.4 + 45.8 + 2 \times 103.5)^{\circ}\text{C} = 95.6^{\circ}\text{C}$$

Node 3, Plane surface with convection, Eq. 4.42,  $T_{m,n} = T_3$

$$2 \left[ \frac{h\Delta x}{k} + 2 \right] T_3 = (2T_{m-1,n} + T_{m,n+1} + T_{m,n-1}) + \frac{2h\Delta x}{k} T_{\infty}$$

$$h\Delta x/k = 50 \text{ W/m}^2 \cdot \text{K} \times 0.1 \text{ m} / 1.5 \text{ W/m} \cdot \text{K} = 3.33$$

$$2(3.33 + 2)T_3 = (2 \times 103.5 + 45.8 + 67.0)^{\circ}\text{C} + 2 \times 3.33 \times 30^{\circ}\text{C}$$

$$T_3 = \frac{1}{10.66}(319.80 + 199.80)^{\circ}\text{C} = 48.7^{\circ}\text{C}$$

(b) The heat rate per unit thickness from the surface to the fluid is determined from the sum of the convection rates from each control volume surface.

$$q'_{\text{conv}} = q'_a + q'_b + q'_c + q'_d$$

$$q'_i = h\Delta y_i (T_i - T_{\infty})$$

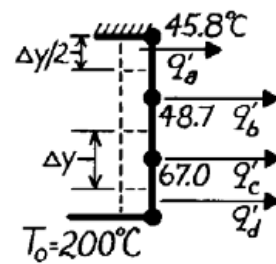
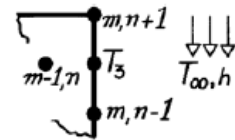
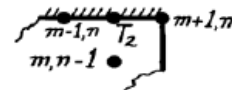
$$q'_{\text{conv}} = 50 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} \left[ \frac{0.1}{2} \text{ m} (45.8 - 30.0)^{\circ}\text{C} + \right.$$

$$0.1 \text{ m} (48.7 - 30.0)^{\circ}\text{C} +$$

$$0.1 \text{ m} (67.0 - 30.0)^{\circ}\text{C} +$$

$$\left. \frac{0.1 \text{ m}}{2} (200.0 - 30.0)^{\circ}\text{C} \right]$$

$$q'_{\text{conv}} = (39.5 + 93.5 + 185.0 + 425) \text{ W/m} = 743 \text{ W/m}$$

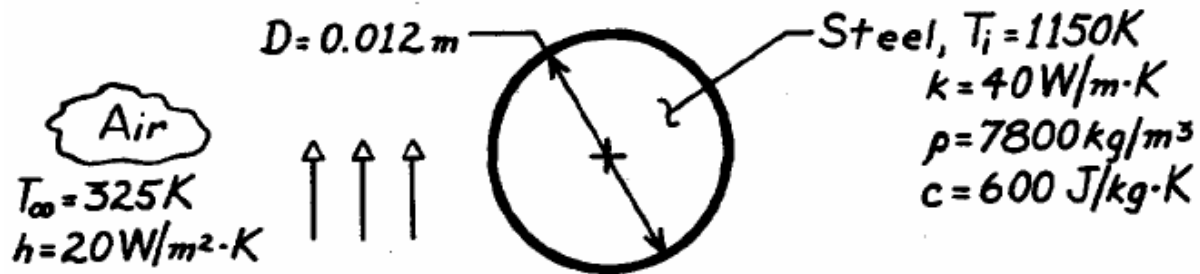


4. Work Incropera & DeWitt Problem 5.5. Ignore cooling by radiation.

**KNOWN:** Diameter and initial temperature of steel balls cooling in air.

**FIND:** Time required to cool to a prescribed temperature.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Negligible radiation effects, (2) Constant properties.

**ANALYSIS:** Applying Eq. 5.10 to a sphere ( $L_c = r_o/3$ ),

$$Bi = \frac{hL_c}{k} = \frac{h(r_o/3)}{k} = \frac{20 \text{ W/m}^2 \cdot \text{K} (0.002 \text{ m})}{40 \text{ W/m} \cdot \text{K}} = 0.001.$$

Hence, the temperature of the steel remains approximately uniform during the cooling process, and the lumped capacitance method may be used. From Eqs. 5.4 and 5.5,

$$t = \frac{\rho V c_p}{h A_s} \ln \frac{T_i - T_\infty}{T - T_\infty} = \frac{\rho (\pi D^3 / 6) c_p}{h \pi D^2} \ln \frac{T_i - T_\infty}{T - T_\infty}$$

$$t = \frac{7800 \text{ kg/m}^3 (0.012 \text{ m}) 600 \text{ J/kg} \cdot \text{K}}{6 \times 20 \text{ W/m}^2 \cdot \text{K}} \ln \frac{1150 - 325}{400 - 325}$$

$$t = 1122 \text{ s} = 0.312 \text{ h}$$

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**COMMENTS:** Due to the large value of  $T_i$ , radiation effects are likely to be significant during the early portion of the transient. The effect is to shorten the cooling time.

5. Work Incropera & DeWitt Problem 5.7.

(b) Using the  $h$  you determined from part (a), determine the Biot numbers for spheres made of stainless steel AISI 304 and soft vulcanized rubber. (Use appendices A.1 and A.3)

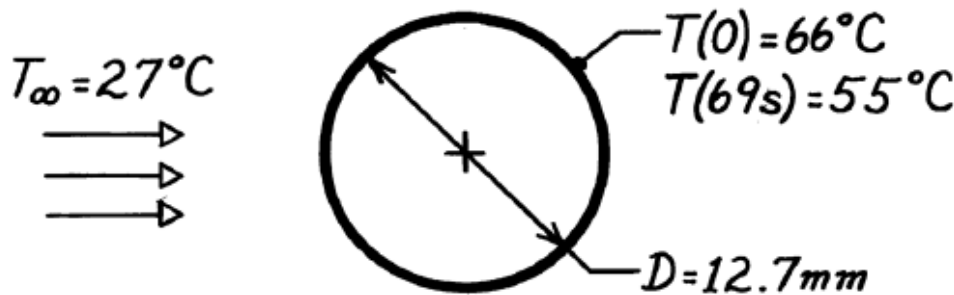
(c) What is the maximum diameter of each sphere (copper, stainless steel AISI 304, soft vulcanized rubber) that would allow the lumped capacitance model to be applied (i.e.,  $Bi = 0.1$ )

**PROBLEM 5.7**

**KNOWN:** The temperature-time history of a pure copper sphere in an air stream.

**FIND:** The heat transfer coefficient between the sphere and the air stream.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Temperature of sphere is spatially uniform, (2) Negligible radiation exchange, (3) Constant properties.

**PROPERTIES:** Table A-1, Pure copper (333K):  $\rho = 8933 \text{ kg/m}^3$ ,  $c_p = 389 \text{ J/kg}\cdot\text{K}$ ,  $k = 398 \text{ W/m}\cdot\text{K}$ .

**ANALYSIS:** The time-temperature history is given by Eq. 5.6 with Eq. 5.7.

$$\frac{\theta(t)}{\theta_i} = \exp\left(-\frac{t}{R_t C_t}\right) \quad \text{where} \quad R_t = \frac{1}{hA_s} \quad A_s = \pi D^2$$

$$C_t = \rho V c_p \quad V = \frac{\pi D^3}{6}$$

$$\theta = T - T_\infty$$

Recognize that when  $t = 69\text{s}$ ,

$$\frac{\theta(t)}{\theta_i} = \frac{(55 - 27)^\circ\text{C}}{(66 - 27)^\circ\text{C}} = 0.718 = \exp\left(-\frac{t}{\tau_t}\right) = \exp\left(-\frac{69\text{s}}{\tau_t}\right)$$

and solving for  $\tau_t$  find

$$\tau_t = 208\text{s}.$$

Hence,

$$h = \frac{\rho V c_p}{A_s \tau_t} = \frac{8933 \text{ kg/m}^3 \left(\frac{\pi (0.0127)^3}{6} \text{ m}^3\right) 389 \text{ J/kg}\cdot\text{K}}{\pi (0.0127)^2 \text{ m}^2 \times 208\text{s}}$$

$$h = 35.3 \text{ W/m}^2 \cdot \text{K}.$$

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**COMMENTS:** Note that with  $L_c = D_o/6$ ,

$$Bi = \frac{hL_c}{k} = 35.3 \text{ W/m}^2 \cdot \text{K} \times \frac{0.0127}{6} \text{ m} / 398 \text{ W/m}\cdot\text{K} = 1.88 \times 10^{-4}.$$

Hence,  $Bi < 0.1$  and the spatially isothermal assumption is reasonable.

- (b) Using the  $h$  you determined from part (a), determine the Biot numbers for spheres made of stainless steel AISI 304 and soft vulcanized rubber. (Use appendices A.1 and A.3)
- (c) What is the maximum diameter of each sphere (copper, stainless steel AISI 304, soft vulcanized rubber) that would allow the lumped capacitance model to be applied (i.e.,  $Bi = 0.1$ )

*(b) using  $h = 35.3 \text{ W/m}^2\text{-K}$ ,  $Bi (Ti) = hL_c/k$ ;*

*$L_c$  is the same as in part (a) =  $r_o/3 = 0.0127 / 6 = 0.00212 \text{ m}$*

*STAINLESS STEEL:*

*$k$  for stainless steel @ 333 K (interpolating from Appendix A.1) =  $15.8 \text{ W/m-K}$*

*$Bi = (35.3 * .00212) / 15.8 = 0.00473$*

*SOFT VULCANIZED RUBBER:*

*$k$  for rubber @ 300K (that's as close as we can get in Appendix A.3) =  $0.13 \text{ W/m-K}$*

*$Bi = (35.3 * .00212) / .13 = 0.58$  (Lumped capacitance would not work!)*

*(c) setting  $Bi = 0.1$ , find  $L_c$  (and  $d$ ) for each material.  $L_c = r_o/3 = d/6$*

*$Bi = h L_c / k = h * d / 6 k$*

*solving for  $d$ :  $d = Bi * 6 * k / h$*

*COPPER:  $d = 0.1 * 6 * 401 / 35.3 = 6.82 \text{ meters! (HUGE!)}$*

*STAINLESS STEEL:  $d = 0.1 * 6 * 15.8 / 35.3 = 0.27 \text{ m (nearly a foot)}$*

*RUBBER:  $d = 0.1 * 6 * 0.13 / 35.3 = 2.21 * 10^{-3} \text{ m (just 2 mm!)}$*

6. A recently fabricated sheet of 5m-by-2m aluminum with thickness 2 mm comes out of the furnace at 400 °C. If the aluminum must reach 100 °C at its center (the midplane) prior to stacking, how long must the sheet be exposed to ambient air (25 °C,  $h = 10 \text{ W/m}^2\text{-K}$ )?

$$k_{\text{Al}} = 237 \text{ W/m-K}$$

$$\rho_{\text{Al}} = 2702 \text{ kg/m}^3$$

$$c_{p, \text{Al}} = 903 \text{ J/kg-K}$$

Here, you need to first check the Biot number to see if you can use lumped capacitance and assume that the midplane temperature is equal to the surface temperature of the aluminum.

$$\text{So, } Bi = hL_c/k$$

where  $L_c = \text{characteristic length} = V/A_s = \{5 * 2 * 0.002 \text{ m}^3\} / \{2 * (5 * 2) \text{ m}^2\} = 0.001 \text{ m}$  (the half-thickness of the sheet)

$$Bi = 10 \text{ W/m}^2\text{-K} * 0.001 \text{ m} / 237 \text{ W/m-K} = 4.2 * 10^{-5}$$

Thus,  $Bi \ll 1$ , and the lumped capacitance model can be used.

This means that  $T_{\text{center}} = T_{\text{surface}}$ .

Here, convection is the limiting mode of heat transfer.

$$T - T_{\text{inf}} / (T_i - T_{\text{inf}}) = \exp\{-hA_s/\rho V c_p\} t\} \quad \{\text{Recall that mass} = \text{density} * \text{volume}\}$$

Plug in values and solve:

$$t = -2702 * (V/A_s = 0.001) * 903 / 10 * \ln((100-25)/(400-25)) = 393 \text{ s}$$

7. For problem #6, if the sheet were made of cork, what would the Biot number be? (the air properties are the same as in #6).

Sketch (qualitatively) the temperature profile across the thickness of the sheets (T vs. x) for three different times during the cooling process. Draw three temperature profiles for (A) aluminum, and on a separate graph three temperature profiles for (B) cork. The three times you select should be:  $t = 0$  (before cooling), and two times during the cooling (but not at infinite time).

Here, look up  $k$  for cork. Density and  $c_p$  also change. All other variables stay the same.

$k = 0.039$  (cork board--- NOT cork loose fill!)

$Bi = hL_c/k = 10 * 0.001 / 0.039 = 0.26$ . Although this is  $< 1$ , it is not sufficient to assume lumped capacitance, so the center of the sample will always be hotter than the surface.

The temperature profiles are flat for aluminum. For cork, the profiles will be concave down, though the actual shape may be different than what is shown.

