ChE 306: HEAT TRANSFER
FALL 2010
Homework #4 Ch 6 & 7 (80 points)
DUE: WEDNESDAY, SEPTEMBER 29
(Appendix Tables A.4 to A.7 will be helpful for the HW set.)

1. A 3-inch diameter titanium sphere at 0 ^oF is dropped into an oil bath at 200 ^oF.

DATA	Titanium sphere	Oil bath
Density (lb_m/ft^3)	281	66.6
Heat Capacity ($Btu/lb_m\text{-}^oF$)	0.125	0.000652
Thermal Conductivity ($Btu/h\text{-}ft\text{-}^oF$)	12.7	0.151

A thermocouple placed in the center of the sphere gives the following data:

Time (min)	**Temperature (^oF)**
0	0
0.5	19
1	36
2	66
5	126
10	173
20	196

What is the convective heat transfer coefficient for the oil?
(Hint: go to the material we covered in Chapter 5)

To solve, would normally first determine the Biot number for the sphere.
Since we don't have h, we can't do this... yet.

So... the easiest way to work this out is to assume Bi < 0.1, and that lumped capacitance can be used.
If this is true, $T\text{-}T_{inf} / (T_{initial} - T_{inf}) = \exp(-hAt/mc_p) = \exp(-hAt/\rho Vc_p)$

In this equation, we have data for T vs. t, we know T inf, T initial, density, heat capacity and can calculate A and V.... or directly find A/V = 4 pi r2 / 4/3 pi r3 = 3/r.

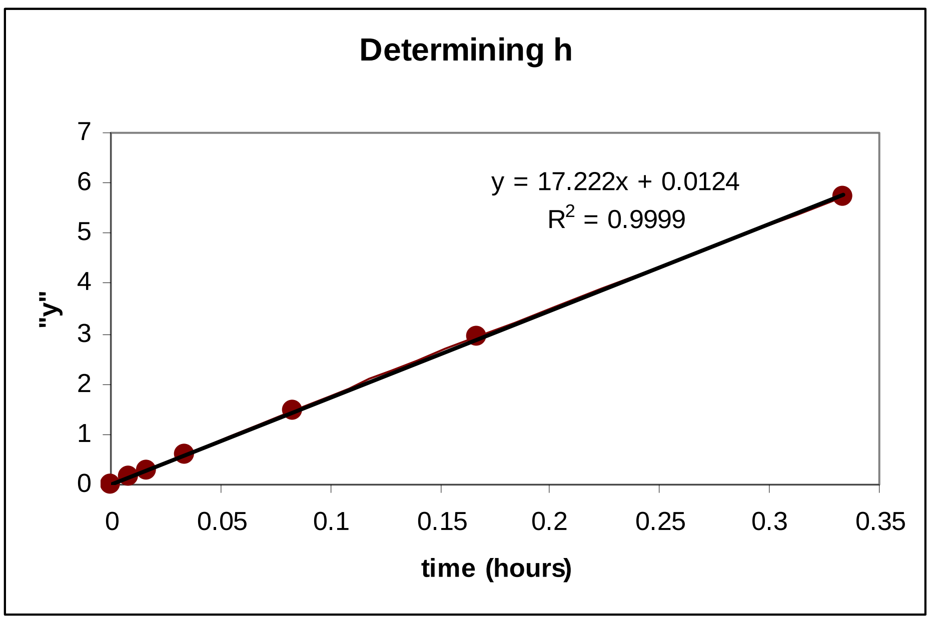
To get a linear plot and find h, take the ln of both sides, and multiply both sides by $-\rho Vc_p/A$ (or $-r\rho c_p/3$) getting
$-\rho c_p r/3 \ln \{(T\text{-}T_{inf})/ (T_{initial} - T_{inf})\} = ht$
$-\rho c_p r/3 = -(281\ lb_m/ft^3) * (0.125\ Btu/lb_m{}^oF) * (1.5/12\ ft) / 3 = -1.46\ Btu/ft^2\text{-}^oF$
Then plot values on the left ("y") vs. time. The slope of the (hopefully) straight line that results will be h.
You can verify this by determining the slope of this line.
Be sure to keep your units straight as you do this.... switch time to HOURS! switch radius to FEET!
and you'll end up with the units of h = $lb_m/ft^3 * Btu/lb_m{}^oF * ft\ / h$ ..... or $Btu/ft^2\text{-}h\text{-}^oF$

There are other ways to do this, but a PLOT is required. The plot takes ALL of the data into consideration to determine h. If you choose individual points, you'll only find an h for that portion of the experiment... it may be close, but the plot gives a much more accurate measure of the whole convection process.

Data from Excel

time (min)	time (h)	Temp oF	T-Tinf/Tinit-Tinf	pcpr/3 =	$-\rho c_p r/3 \ln \{(T-T_{inf})/(T_{initial} - T_{inf})\}$
0	0	0	1	1.464	0
0.5	0.008333	19	0.905		0.146136971
1	0.016667	36	0.82		0.290532174
2	0.033333	66	0.67		0.586299157
5	0.083333	126	0.37		1.455585328
10	0.166667	173	0.135		2.931631453
20	0.333333	196	0.02		5.72720168

PLOT:



Thus, **<u>h = 17.22 Btu/ft^2-h-^{o}F (or 17.28 if you force the line through the origin)</u>**

Now, let's check to be certain that our assumption of using lumped capacitance was valid:

$Bi = hL_c/k_s$ = (17.22 Btu/ft^2-h-^{o}F) (1.5 in/3 * 1 ft/12 in) / (12.7 Btu/h-ft-^{o}F) = 0.056

YES! Method is OK, answer is OK.

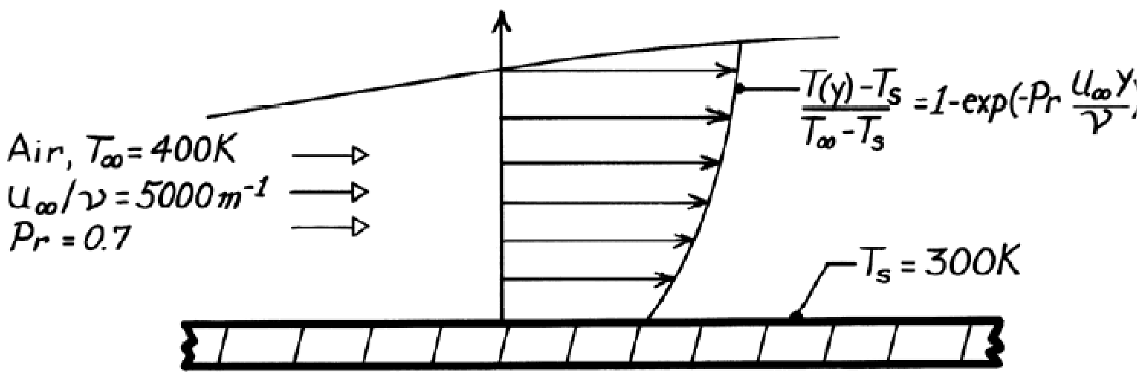
2. Work Incropera and DeWitt Problem 6.3.

PROBLEM 6.3

KNOWN: Boundary layer temperature distribution.

FIND: Surface heat flux.

SCHEMATIC:



PROPERTIES: *Table A-4*, Air (T_s = 300K): k = 0.0263 W/m·K.

ANALYSIS: Applying Fourier's law at y = 0, the heat flux is

$$q''_s = -k\frac{\partial T}{\partial y}\bigg|_{y=0} = -k\left(T_\infty - T_s\right)\left[Pr\frac{u_\infty}{\nu}\right]\exp\left[-Pr\frac{u_\infty y}{\nu}\right]\bigg|_{y=0}$$

$$q''_s = -k\left(T_\infty - T_s\right)Pr\frac{u_\infty}{\nu}$$

$$q''_s = -0.0263\ \text{W/m}\cdot\text{K}\left(100\text{K}\right)0.7\times 5000\ 1/\text{m}.$$

$$q''_s = -9205\ \text{W/m}^2.$$ <

COMMENTS: (1) Negative flux implies convection heat transfer to the surface.

(2) Note use of k at T_s to evaluate q''_s from Fourier's law.

3. A tunnel is designed to test different fluids flowing over a flat plate. The turbine can generate air speeds of up to 50 m/s. You wish to study boundary layer formation for Reynolds numbers (Re_x) up to 10^8.

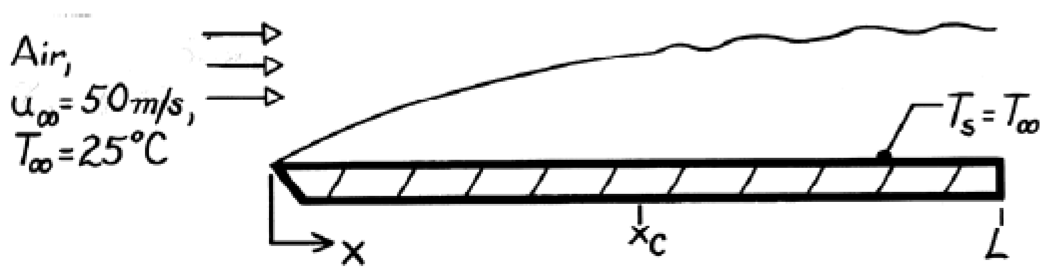
A. How long must the flat plate be to achieve this Re_x value for air at 25 °C?

B. At what x-position along the plate does the flow change from laminar to turbulent? ($Re_{x,c} = 5 \times 10^5$)

KNOWN: Air speed and temperature in a wind tunnel.

FIND: (a) Minimum plate length to achieve a Reynolds number of 10^8, (b) Distance from leading edge at which transition would occur.

SCHEMATIC:



ASSUMPTIONS: (1) Isothermal conditions, $T_s = T_\infty$.

PROPERTIES: *Table A-4*, Air (25°C = 298K): $\nu = 15.71 \times 10^{-6} m^2/s$.

ANALYSIS: (a) The Reynolds number is

$$Re_x = \frac{\rho u_\infty x}{\mu} = \frac{u_\infty x}{\nu}.$$

To achieve a Reynolds number of 1×10^8, the minimum plate length is then

$$L_{min} = \frac{Re_x \nu}{u_\infty} = \frac{1\times10^8 \left(15.71\times10^{-6} m^2/s\right)}{50 \text{ m/s}}$$

$$L_{min} = 31.4 \text{ m.}$$ <

(b) For a transition Reynolds number of 5×10^5

$$x_c = \frac{Re_{x,c} \nu}{u_\infty} = \frac{5\times10^5 \left(15.71\times10^{-6} m^2/s\right)}{50 \text{ m/s}}$$

$$x_c = 0.157 \text{ m.}$$ <

COMMENTS: Note that

$$\frac{x_c}{L} = \frac{Re_{x,c}}{Re_L}$$

4. The same tunnel from problem 3 is used to test different fluids. For each of the situations below, find the distance from the front edge of the plate where a fluid flowing at 1 m/s parallel to the surface will change from laminar to turbulent flow. ($Re_{x,c}$ for a flat plate is 5 * 10^5).

A. Air at 25 °C, B. Air at 70 °C, C. Carbon Dioxide at 25 °C,

D. Water at 25 °C, E. Engine oil at 25 °C, F. Mercury at 25 °C

(Note: You may approximate physical properties at 25 °C to be those for 300 K and 70°C/340K)

Much of this problem is solved as at right (parts A,B,D and E):

x_c varies with kinematic viscosity:

A. x_c = 7.95 m

B. x_c = 10.5 m

C. CO_2
look up properties of saturated water in Table A.6
kinematic viscosity = $8.40 * 10^{-6}$ m^2/s
5*105 = 1m/s * x_c / ($8.40*10^{-6}$ m^2/s)
x_c = 4.2 m

D. WATER:
look up properties of saturated water in Table A.6
viscosity = 855 * 10^{-6} Ns/m^2
specific volume = 1.003 * 10^{-3} m^3/kg
kinematic visc = visc * spec. vol
= 0.858 * 10^{-6} m^2/s
x_c = 0.43 m

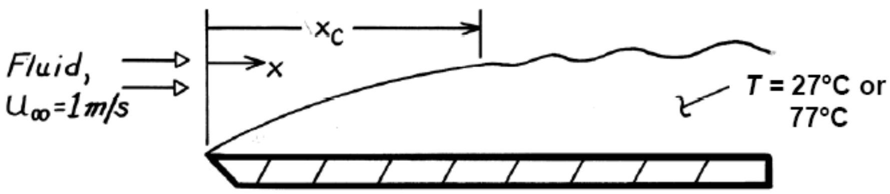
E. x_c = 275 m

F. x_c = 0.056 m

KNOWN: Transition Reynolds number. Velocity and temperature of atmospheric air, engine oil, and mercury flow over a flat plate.

FIND: Distance from leading edge at which transition occurs for each fluid.

SCHEMATIC:



ASSUMPTIONS: Transition Reynolds number is $Re_{x,c} = 5 \times 10^5$.

PROPERTIES: For the fluids at T = 300 K and 350 K:

Fluid	*Table*	$\nu(m^2/s)$ T = 300 K	$\nu(m^2/s)$ T = 350 K
Air (1 atm)	A-4	15.89×10^{-6}	20.92×10^{-6}
Engine Oil	A-5	550×10^{-6}	41.7×10^{-6}
Mercury	A-5	0.1125×10^{-6}	0.0976×10^{-6}

ANALYSIS: The point of transition is

$$x_c = Re_{x,c} \frac{\nu}{u_\infty} = \frac{5 \times 10^5}{1 \text{ m/s}} \nu.$$

Substituting appropriate viscosities, find

Fluid	$x_c(m)$ T = 300 K	$x_c(m)$ T = 350 K
Air	7.95	10.5
Oil	275	20.9
Mercury	0.056	0.049

<

COMMENTS: (1) Note the great disparity in transition length for the different fluids. Due to the effect which viscous forces have on attenuating the instabilities which bring about transition, the distance required to achieve transition increases with increasing ν. (2) Note the temperature-dependence of the transition length, in particular for engine oil. (3) As shown in Example 6.4, the variation of the transition location can have a significant effect on the average heat transfer coefficient associated with convection to or from the plate.

5. Four experimental fluids are tested in parallel flow at 1.0 m/s over a flat plate where the plate temperature is 49 ^{o}C and the fluids are at 5 ^{o}C. Fluids: Air, Water, Engine Oil, Mercury

A. For each fluid, determine the hydrodynamic (velocity) boundary layer thickness at a distance 5 cm from the front edge of the plate.
B. For each fluid, determine the thermal boundary layer thickness at the same distance (5 cm).
C. How would your answer change if the plate was at 10 ^{o}C and the fluids were at 44 ^{o}C?

This problem uses mostly the same fluids as problem 4. The properties are evaluated at the film temperature of (49 + 5) / 2 = 27 C (300K).

A. Table 7.9 at the end of Chapter 7 is very helpful here with summarizing correlations.
First check whether the fluid is laminar or turbulent at the 50 mm distance (this is less than 2 inches, so laminar is probably correct for all 4 fluids):

If it is laminar ($Re < 5 * 10^5$), use equation 7.19 to find the hydrodynamic boundary layer thickness.
If it is turbulent ($5 * 10^5 < Re < 10^8$), use equation 7.35.
The solution to problem 3 provides enough information. The critical transition for each of these fluids is greater than 50 mm (it is only 56 mm for mercury, but at 50 mm, the flow will still be laminar).

B. Since all are laminar, the correlation for flow over a flat plate (Eqn 7.24) can be used for the thermal boundary layer thickness.

Fluid	L (m)	v (m/s)	ν (m^2/s)	Re = vL/ν	Pr	δ_h (m)	δ_t (m)
Air	0.050	1.0	$15.89 * 10^{-6}$	3146	0.707	*0.0045*	*0.0050*
Water	0.050	1.0	$0.858 * 10^{-6}$	58,275	5.83	*0.00103*	*0.00058*
Engine Oil	0.050	1.0	$550 * 10^{-6}$	90.9	6400	*0.026*	*0.0014*
Mercury	0.050	1.0	$0.1125 * 10^{-6}$	444,000	0.181	*0.00038*	*0.00066*

C. Since the film temperature is the same here, there would be no change in the answers to parts A or B.

6.Convective cooling is largely responsible for cooling car engines to prevent overheating. This is especially a problem in slow-moving traffic, and the engine is idling while the car speed is low. Assuming that the only available method to dissipate heat from the engine is convection, **determine the minimum speed (in mph) a driver must go to avoid overheating.** (Note: the inside back cover of your text has useful conversion factors)

Assume that the engine can be represented accurately as a flat plate with square dimensions of 2m by 2m (length by width), with flow over the topside only. The engine surface maintains a constant surface temperature of 90 ^oC, and is cooled by airflow, which is the same speed as the car moves.

The heat generated in the engine (which must be dissipated by convection) is a function of car speed:

$$\mathbf{q = 1000 + 75\ v^{1/2}} \text{ (where v in m/s, and q is in W)}$$

As a worst-case scenario, use the outside air temperature for a warm Alabama summer day: 100 ^oF (38 ^oC). To assist with selection of a Nu correlation, assume that the speed will be below 5 m/s (11.25 mph).

	Air properties at 337 K	Air properties at 363 K
Thermal Conductivity, k	0.029 W/m-K	0.031 W/m-K
Prandtl Number	0.702	0.697
Kinematic Viscosity, ν	$20 * 10^{-6}$ m^2/s	$23 * 10^{-6}$ m^2/s
Density, ρ	1.05 kg/m^3	0.97 kg/m^3
Heat capacity, c_p	1.008 kJ/kg-K	1.011 kJ/kg-K
Viscosity, μ	$203 * 10^{-7}$ $N\text{-}s/m^2$	$215 * 10^{-7}$ $N\text{-}s/m^2$

Here Energy in = Energy Out.

$q_{conv} = q_{engine} = 1000 + 75 * v^{1/2}$

$q_{conv} = h A (T_s - T_\infty) = h * 4m^2 * (90\ ^oC - 38\ ^oC)$

Must find a correlation for h, then solve for v.

Step 1: Is the flow over the engine turbulent or laminar?
With a speed of 5 m/s, need to know air properties:
Film Temperature = (90 + 38) /2 = 64 C or 337 K
ν, k_f, and Pr are given in the left column above.

Since we assume v < 5 m/s the maximum Re = vL/ ν = 5 m/s * 2m / $20 * 10^{-6}$ m^2/s = $5 * 10^5$
Therefore, the flow is laminar over the entire engine and only becomes turbulent as it leaves.
Nusselt # correlation needed for laminar flow over a flat plate:
Eqn 7.30: Nu (average) = $0.664\ Re^{1/2} Pr^{1/3}$ =
Since v is not known, leave v in the Re #: Nu = $0.664 * (v * 2m / 20*10^{-6}\ m^2/s)^{1/2} * 0.702^{1/3} = 186.6\ v^{1/2}$
$h = k_f/L * Nu = (0.029\ W/m\text{-}K / 2m) * 186.6\ v^{1/2} = 2.71\ v^{1/2}\ W/m^2\text{-}K$
Plug into equation with engine velocity:
$1000 + 75 * v^{1/2} = 2.71\ v^{1/2} * 4m^2 * (90\ ^oC - 38\ ^oC) = 563\ v^{1/2}$
v = (1000/(563-75))^2 = **4.20 m/s** or 9.45 mph.
(at this speed the engine produces as much heat as is convected away)

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7. A 20-m long cylindrical pipe with outer diameter of 12 cm is made of aluminum (k = 237 W/m-K; wall thickness 0.2 cm) and is cooled by external forced convection using 27 ^oC air at 2.4 m/s. The pipe's outer surface temperature is 227 ^oC. *Fluid properties are included on the last page of the exam.*

Which air flow orientation (**cross-flow or parallel flow**) will result in a higher cooling rate?
Justify your answer by showing the numerical ratio of the cooling rates in cross flow to parallel flow.

Cross Flow

$q_c = h_c A (T_s - T_\infty)$

since A + $(T_s - T_\infty)$ are the same, compare h's.

$h_c = \frac{Nu_c k_f}{D}$

$Re_D = \frac{VD}{\nu}$ properties @ $T_f = 400K$.

$= \frac{(2.4\ m/s)(0.12\ m)}{26.41 \times 10^{-6}\ m^2/s} = 10{,}905$

Turbulent.

3 possible Nu correlations

$Nu_D = C\ Re^m Pr^{1/3}$ $C = .193$, $m = .618$ (Table 7.2)

$Nu = .193(10{,}905)^{.618}(0.69)^{1/3}$

$Nu_D = 53.3$

$h_c = \frac{(53.3)(.0338\ W/m\text{-}K)}{(.12\ m)}$

$h_c = 15.0\ W/m^2\text{-}K$

All else being the same, Cross-flow will cool 3.47 times faster.

Parallel Flow

$q_p = h_p A (T_s - T_\infty)$

$h_p = \frac{Nu_p k_f}{L}$

$Re = \frac{VL}{\nu}$

$= \frac{(2.4\ m/s)(20\ m)}{(26.41 \times 10^{-6}\ m^2/s)}$

$= 1.82 \times 10^6$ turbulent.

$x_c = \frac{(Re_{x_c})\nu}{V}$

$= \frac{(5\times10^5)(26.41\times10^{-6}\ m^2/s)}{(2.4\ m/s)}$

$= 5.5\ m$

Needs mixed correlation.

$Nu = (.037 Re_L^{4/5} - 871) Pr^{1/3}$

$Nu_L = 2{,}557$

$h_p = \frac{(2557)(.0338)}{20\ m}$

$h_p = 4.32$

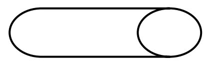
For the cross-flow solution, two other correlations will work.

If you used Eq. 7.53, you must determine fluid properties at Tinf (and Pr at the surface temp).
This results in a different Re # = 18,100 because of the lower kinematic viscosity
Determine C & m from Table 7.4 to be 0.26 and 0.6. n = 0.37 for fluids with Pr < 10 (p.427),
so Nu = 82.76
using k_f at Tinf, h is then 18.1 W/m2-K, and the q-ratio is even higher (4.2 times faster for cross flow)

If you used Eq. 7.54, all properties are at Tfilm, so Re is 10905 again. Pr = 0.69
Plug in and get Nu = 55.6, which results in h = 15.7 W/m2-K. and the q-ratio is 3.63

8. Airplanes can be modeled as circular tubes flying in high altitudes through very cold atmospheric conditions. It is important that there is enough power on board to keep the cabin temperature normalized during flight. For the conditions below, **determine the power (in W) needed for a heater inside the plane to balance the heat lost through convection to the upper atmosphere.** Ignore the wings, tail, and nose cone of the plane, and only consider the air moving parallel to the tubular cabin.

Plane dimensions
Outside Diameter: 10 m
Length: 60 m
Wall thickness: 0.1 m



Internal cabin air temperature: 25 °C
External (high altitude) temperature: -40 °C
Internal convective heat transfer coefficient (h_i) = 7.0 W/m^2-K
Airspeed: 150 mph (= 67 m/s)
Thermal resistance of the shell (walls): 0.00026 K/W

You may use -23 °C as the film temperature to determine the properties of the outside air.

Here, the airplane is modeled as a cylinder. The wind is parallel to the surfaces of the fuselage, so it can be treated as a flat plate for determining heat transfer correlations.

Here, we know two temperatures, inside air and outside air.
Between those extremes, there are three resistances:
1. internal h (given as 7.0 W/m^2-K)
2. conductivity of the wall (R_{wall} given above)
3. external/outside h, which is not given and must be found by correlation.

Once the three resistances are found, (From Figure 3.6):
$q = dT/\Sigma R = \{25 - (-40)\} / \{1/h_iA_i + \ln(r_2/r_1) / (2 \text{ pi } L\, k) + 1/h_oA_o\}$
Unknowns: h_o & k, BUT R_{wall} is given, so the middle resistance term = 0.00026K/W
External h:
must determine Re = vL/nu = 67 m/s * 60 m / 11.44 * 10^{-6} m^2/s = 3.51 * 10^8
Flow is turbulent when it leaves the back surface of the plane.
Do we need a mixed or turbulent correlation?
Find L for Re = 5 * 10^5
L = 5 * 10^5 * 11.44 * 10^{-6} / 60 = 0.095 m (less than 5 % of the plane length).
Thus, use a turbulent correlation.
Local or average Nu? (doesn't matter for turbulent- they're the same)
There are no correlations for Re > 10^8, so the best we can do is choose eqn. 7.36
$Nu = 0.0296 Re^{4/5} Pr^{1/3}$
Pr (for air at 250 K) is 0.72
Thus, Nu = 0.0296 * (3.51*10^8)$^{4/5}$ * 0.72$^{1/3}$ = 1.82 * 10^5
h = Nu * kf / L = 1.82 * 10^5 * 0.0223 W/mK / 60 m = 67.7 W/m^2-K
Now you have all of the numbers needed to solve for q.
Ai = pi * Di * L = pi * 9.8m * 60 m = 1847 m^2
Ao = pi * Do * L = pi * 10m * 60 m = 1885 m^2
q = 65K / {1/(7*1847) + .00026 + 1/(67.7*1885)} = **1.88 * 10^5 W or 188kW**