ChE 306: HEAT TRANSFER
FALL 2010: Homework #6
Ch 9 & 10: Free Convection & Boiling/Condensation
(70 points)
DUE: WEDNESDAY, OCTOBER 26

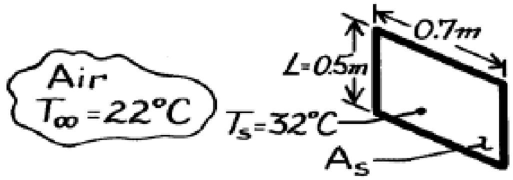
1. Work Incropera and DeWitt Problem 9.13

PROBLEM 9.13

KNOWN: Oven door with average surface temperature of 32°C in a room with ambient air at 22°C.

FIND: Heat loss to the room. Also, find effect on heat loss if emissivity of door is unity and the surroundings are at 22°C.

SCHEMATIC:



ASSUMPTIONS: (1) Ambient air is quiescent, (2) Surface radiation effects are negligible.

PROPERTIES: *Table A-4*, Air (T_f = 300K, 1 atm): $\nu = 15.89 \times 10^{-6}\ m^2/s$, k = 0.0263 W/m·K, $\alpha = 22.5 \times 10^{-6}\ m^2/s$, Pr = 0.707, $\beta = 1/T_f = 3.33 \times 10^{-3}\ K^{-1}$.

ANALYSIS: The heat rate from the oven door surface by convection to the ambient air is

$$q = \bar{h} A_s (T_s - T_\infty) \qquad (1)$$

where $\bar{h}$ can be estimated from the free-convection correlation for a vertical plate, Eq. 9.26,

$$\overline{Nu}_L = \frac{\bar{h} L}{k} = \left\{ 0.825 + \frac{0.387\, Ra_L^{1/6}}{\left[1 + (0.492/Pr)^{9/16}\right]^{8/27}} \right\}^2 . \qquad (2)$$

The Rayleigh number, Eq. 9.25, is

$$Ra_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha} = \frac{9.8\,m/s^2\,(1/300K)(32-22)K \times 0.5^3 m^3}{15.89\times10^{-6} m^2/s \times 22.5\times10^{-6} m^2/s} = 1.142\times10^8.$$

Substituting numerical values into Eq. (2), find

$$\overline{Nu}_L = \left\{ 0.825 + \frac{0.387\left(1.142\times10^8\right)^{1/6}}{\left[1 + (0.492/0.707)^{9/16}\right]^{8/27}} \right\}^2 = 63.5$$

$$\bar{h}_L = \frac{k}{L}\overline{Nu}_L = \frac{0.0263\,W/m\cdot K}{0.5m} \times 63.5 = 3.34\,W/m^2\cdot K.$$

The heat rate using Eq. (1) is

$$q = 3.34\,W/m^2\cdot K \times (0.5\times0.7)m^2 (32-22)K = 11.7\,W. \qquad <$$

Heat loss by radiation, assuming ε = 1, is

$$q_{rad} = \varepsilon A_s \sigma \left(T_s^4 - T_{sur}^4\right)$$

$$q_{rad} = 1(0.5\times0.7)m^2 \times 5.67\times10^{-8} W/m^2\cdot K^4 \left[(273+32)^4 - (273+22)^4\right] K^4 = 21.4\,W. \qquad <$$

Note that heat loss by radiation is nearly double that by free convection. Using Eq. (1.9), the radiation heat transfer coefficient is $h_{rad} = 6.4\ W/m^2\cdot K$, which is twice the coefficient for the free convection process.

2. Verify the following news report as true or false (you must show numerical work to get credit):

Hypothermia research at the University of North Dakota claims that a person who falls into cold (stagnant) water at 10 °C will experience 30 times the heat loss rate as a person standing in air at 10 °C. Model the person as a nude, vertical cylinder. T_{skin} = 34 °C.

Note: There are other possible solutions here, but you have to make several assumptions: how tall is the person: are they best modeled as a cylinder or a sphere, if a cylinder, are they standing or lying down? Your answers should be similar to below, but maybe not exactly the same... plus, I disagree with the assumption that a person has a surface temperature of 25 °C...

KNOWN: Person, approximated as a cylinder, experiencing heat loss in water or air at 10°C.

FIND: Whether heat loss from body in water is 30 times that in air.

ASSUMPTIONS: (1) Person can be approximated as a vertical cylinder of diameter D = 0.3 m and length L = 1.8 m, at 25°C, (2) Loss is only from the lateral surface.

PROPERTIES: *Table A-4*, Air $\left(\overline{T} = (25+10)^\circ C/2 = 290K, 1\,atm\right)$: k = 0.0293 W/m·K, ν = 19.91 × 10^{-6} m^2/s, α = 28.4 × 10^{-6} m^2/s; *Table A-6*, Water (290K): k = 0.598 W/m·K, $\nu = \mu v_f$ = 1.081 × 10^{-6} m^2/s, $\alpha = k v_f/c_p$ = 1.431 × 10^{-7} m^2/s, β_f = 174 × 10^{-6} K^{-1}.

ANALYSIS: In both water (wa) and air (a), the heat loss from the lateral surface of the cylinder approximating the body is

$$q = \overline{h}\pi DL(T_s - T_\infty)$$

where T_s and T_∞ are the same for both situations. Hence,

$$\frac{q_{wa}}{q_a} = \frac{\overline{h}_{wa}}{\overline{h}_a}$$

Vertical cylinder in air:

$$Ra_L = \frac{g\beta\Delta TL^3}{\nu\alpha} = \frac{9.8\,m/s^2 \times (1/290\,K)(25-10)K(1.8m)^3}{19.91\times10^{-6}m^2/s \times 28.4\times10^{-6}m^2/s} = 5.228\times10^9$$

Using Eq. 9.24 with C = 0.1 and n = 1/3,

$$\overline{Nu}_L = \frac{\overline{h}_L L}{k} = CRa_L^n = 0.1\left(5.228\times10^9\right)^{1/3} = 173.4 \qquad \overline{h}_L = 2.82\ W/m^2\cdot K.$$

Vertical cylinder in water:

$$Ra_L = \frac{9.8\,m/s^2 \times 174\times10^{-6}K^{-1}(25-10)K(1.8\,m)^3}{1.081\times10^{-6}m^2/s \times 1.431\times10^{-7}\,m^2/s} = 9.643\times10^{11}$$

Using Eq. 9.24 with C = 0.1 and n = 1/3,

$$\overline{Nu}_L = \frac{\overline{h}L}{k} = CRa_L^n = 0.1\left(9.643\times10^{11}\right)^{1/3} = 978.9 \qquad \overline{h}_L = 328\ W/m^2\cdot K.$$

Hence, from this analysis we find

$$\frac{q_{wa}}{q_a} = \frac{328\ W/m^2\cdot K}{2.8\ W/m^2\cdot K} = 117$$

which compares poorly with the claim of 30.

COMMENTS: In air, radiation would contribute significantly to the heat loss. Assuming $\varepsilon = 1$, $h_{rad} = \sigma\varepsilon(T_s^4 + T_{sur}^4)/(T_s + T_{sur}) = 5.6\ W/m^2\cdot K$. Thus, $h_{a,tot} = \overline{h}_a + h_{rad} = 8.4 W/m^2\cdot K$ and q_{wa}/q_a = 328/8.4 = 39. This is much closer to the claim of 30 times.

2. (cont'd):

A little more accurately, for a film temperature of 298K (it should be 295 K based on (34 + 10)/ 2 = 22 C, but close enough....

Using these temperatures:
For Air

$Ra = 6.97 \times 10^9$
$Nu_L = 191$
$h_L = 3.11\ W/m^2\text{-}K$

For Water:

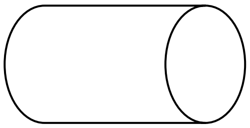
$Ra = 1.29 * 10^{12}$
$Nu_L = 1087$
$h_L = 361\ W/m^2\text{-}K$

Since q = h A (Ts - Tinf), the ratio of h's is adequate to compare heat loss to the air vs. to the water.

The rate of heat loss is over 100 times faster in the cold water, so the report underestimates the difference between air and water.

3. A continuous crystallization process is carried out by passing an aqueous salt solution at 90 °C through a large cylindrical horizontal tube, and collecting the product at 30 °C. There is no additional mixing in the reactor, but the fluid passes through the tube at 45 cm/s.

CRYSTALLIZING TUBE
Length = 10 m; Diameter is 1 m.
Surface temperature of walls = 15 °C.



SALT SOLUTION (assume constant)
Density = 0.989 kg/m^3
Thermal conductivity = 0.657 W/m-K
Heat capacity = 4189 J/kg-K
Prandtl Number = 2.81
Kinematic Viscosity = 4.73 x 10^{-4} m^2/s
Volumetric Thermal Expansion Coefficnt = 543 x 10^{-6} K^{-1}
Thermal Diffusivity = 1.59 x 10^{-4} m^2/s
Viscosity = 468 x 10^{-6} $N\text{-}s/m^2$

a. What **temperature** would be used to estimate the physical properties of the salt solution?

b. Using Gr_D and Re_D, determine if the heat transfer from the salt solution to the tank walls is by **free or forced convection, or both**?

c. Using the known change in internal energy of the fluid and log-mean temperature difference for the heat transfer equation, **estimate the average convective heat transfer coefficient, h**, for convection heat transfer with the tank walls. (Note: Nusselt number correlations are not necessary)

A. Use the mean fluid temperature... since it warms in the tube, the entire fluid is part of the boundary layer and the properties are best estimated as = (90 + 30) /2 = 60 °C.

B. Must compare Gr_D/Re_D^2 to 1.

$Gr_D = g\beta (T_s - T_\infty) D^3 / \nu^2$ (assume T_∞ is approx. Tmean)
$= 9.8\ m/s^2 * 543 * 10^{-6}\ K^{-1}) * 45\ K * (1m)^3 / (4.73 * 10^{-4}\ m^2/s)^2$
$Gr_D = 1.07 * 10^6$

$Re_D = vD/\nu = (0.45\ m/s) * (1m) / (4.73 * 10^{-4}\ m^2/s)$
$Re_D = 951$

$Gr_D/Re_D^2 = 1.18$.
Since this is close to 1, both free and forced convection are important for heat transfer.

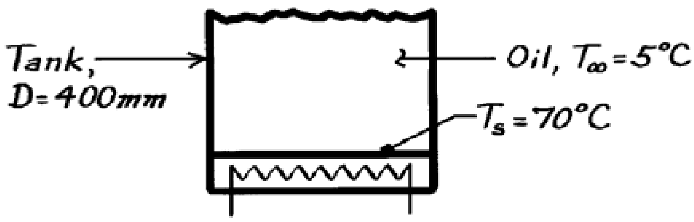
C. $q = m\, c_p (T_o - T_i) = h\, A\, \Delta T_{lm}$
(NOTE: YOU DON"T NEED TO USE ANY CORRELATIONS!)
Everything is known in this equation except h.
$h = m\, c_p (T_o - T_i)/A_s (T_o - T_i) / \ln (T_s - T_i)/(T_s - T_o)$
(NOTE: LOG MEAN TEMP DIFFERENCE REQUIRED: CONSTANT SURFACE TEMP, WITH THE FLUID COOLING AS IT GOES THROUGH THE PIPE!)
Get mass flow rate from velocity, density and cross-sectional area = 0.035 kg/s
Solving for h = 75.1 W/m^2-K

4. A disk-shaped horizontal heater is placed below a large tank filled with engine oil. The disk and tank both have a diameter of 400 mm, but the tank is much taller. The bulk oil temperature is 5 °C. **Estimate the power (W) required** for the heater to maintain the surface temperature at the bottom of the tank at 70 °C. (You may ignore the thermal resistance of the metal tank.) Use the characteristic length $L_c = A_s/P$ for a horizontal flat plate.

KNOWN: Electric heater at bottom of tank of 400mm diameter maintains surface at 70°C with engine oil at 5°C.

FIND: Power required to maintain 70°C surface temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Oil is quiescent, (2) Quasi-steady state conditions exist.

PROPERTIES: *Table A-5*, Engine Oil ($T_f = (T_\infty + T_s)/2 = 310K$): $\nu = 288 \times 10^{-6}\ m^2/s$, $k = 0.145\ W/m\cdot K$, $\alpha = 0.847 \times 10^{-7}\ m^2/s$, $\beta = 0.70 \times 10^{-3}\ K^{-1}$.

ANALYSIS: The heat rate from the bottom heater surface to the oil is

$$q = \bar{h} A_s (T_s - T_\infty)$$

where $\bar{h}$ is estimated from the appropriate correlation depending upon the Rayleigh number Ra_L, from Eq. 9.25, using the characteristic length, L, from Eq. 9.29,

$$L = \frac{A_s}{P} = \frac{\pi D^2/4}{\pi D} = \frac{D}{4} = \frac{0.4m}{4} = 0.1m.$$

The Rayleigh number is

$$Ra_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha}$$

$$Ra_L = \frac{9.8m/s^2 \times 0.70 \times 10^{-3} K^{-1}(70-5)K \times 0.1^3 m^3}{288 \times 10^{-6} m^2/s \times 0.847 \times 10^{-7} m^2/s} = 1.828 \times 10^7.$$

The appropriate correlation is Eq. 9.31 giving

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = 0.15 Ra_L^{1/3} = 0.15\left(1.828 \times 10^7\right)^{1/3} = 39.5$$

$$\bar{h} = \frac{k}{L}\overline{Nu}_L = \frac{0.145\ W/m\cdot K}{0.1m} \times 39.5 = 57.3\ W/m^2\cdot K.$$

The heat rate is then

$$q = 57.3\ W/m^2\cdot K(\pi/4)(0.4m)^2(70-5)K = 468\ W.$$ <

COMMENTS: Note that the characteristic length is D/4 and not D; however, A_s is based upon D. Recognize that if the oil is being continuously heated by the plate, T_∞ could change. Hence, here we have analyzed a quasi-steady state condition.

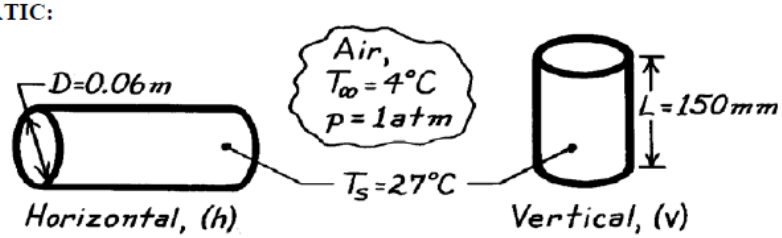
5. Two cans of your favorite beverage are initially at room temperature (27 °C) and are placed in the refrigerator at 4 °C: one standing up, and the other laid horizontal. Which will cool to a drinkable temperature fastest?

To get an approximate answer, you may ignore the circular ends of the cans, and only calculate heat loss by free convection from the curved sides; you may also ignore any conduction from the can to the shelf (i.e., assume the cans levitate without touching a surface). The cans may be approximated as cylinders with D = 60 mm and L = 150 mm.

KNOWN: Dimensions and temperature of beer can in refrigerator compartment.

FIND: Orientation which maximizes cooling rate.

SCHEMATIC:



ASSUMPTIONS: (1) End effects are negligible, (2) Compartment air is quiescent, (3) Constant properties.

PROPERTIES: *Table A-4*, Air (T_f = 288.5K, 1 atm): $\nu = 14.87 \times 10^{-6}\ m^2/s$, k =0.0254 W/m·K, $\alpha = 21.0 \times 10^{-6}\ m^2/s$, Pr = 0.71, $\beta = 1/T_f = 3.47 \times 10^{-3}\ K^{-1}$.

ANALYSIS: The ratio of cooling rates may be expressed as

$$\frac{q_v}{q_h} = \frac{\bar{h}_v}{\bar{h}_h}\frac{\pi DL}{\pi DL}\frac{(T_s - T_\infty)}{(T_s - T_\infty)} = \frac{\bar{h}_v}{\bar{h}_h}.$$

For the *vertical* surface, find

$$Ra_L = \frac{g\beta(T_s - T_\infty)}{\nu\alpha}L^3 = \frac{9.8\,m/s^2 \times 3.47\times10^{-3}\,K^{-1}(23°C)}{\left(14.87\times10^{-6}m^2/s\right)\left(21\times10^{-6}m^2/s\right)}L^3 = 2.5\times10^9 L^3$$

$$Ra_L = 2.5\times10^9(0.15)^3 = 8.44\times10^6,$$

and using the correlation of Eq. 9.26,
$$\overline{Nu}_L = \left\{0.825 + \frac{0.387\left(8.44\times10^6\right)^{1/6}}{\left[1+(0.492/0.71)^{9/16}\right]^{8/27}}\right\}^2 = 29.7.$$

Hence
$$\bar{h}_L = \bar{h}_v = \overline{Nu}_L\frac{k}{L} = 29.7\frac{0.0254\,W/m\cdot K}{0.15m} = 5.03\,W/m^2\cdot K.$$

For the *horizontal* surface, find
$$Ra_D = \frac{g\beta(T_s - T_\infty)}{\nu\alpha}D^3 = 2.5\times10^9(0.06)^3 = 5.4\times10^5$$

and using the correlation of Eq. 9.34,
$$\overline{Nu}_D = \left\{0.60 + \frac{0.387\left(5.4\times10^5\right)^{1/6}}{\left[1+(0.559/0.71)^{9/16}\right]^{8/27}}\right\}^2 = 12.24$$

$$\bar{h}_D = \bar{h}_h = \overline{Nu}_D\frac{k}{D} = 12.24\frac{0.0254\,W/m\cdot K}{0.06m} = 5.18\,W/m^2\cdot K.$$

Hence
$$\frac{q_v}{q_h} = \frac{5.03}{5.18} = 0.97.$$ <

COMMENTS: In view of the uncertainties associated with Eqs. 9.26 and 9.34 and the neglect of end effects, the above result is inconclusive. The cooling rates are approximately the same.

6. Water at its saturation point is heated in a small cylindrical copper tank that has a diameter of 15 cm. At atmospheric pressure, the bottom surface of the tank is kept at 115 ^{o}C.

A. How much energy must be added to the pan to boil the water? (Watts)

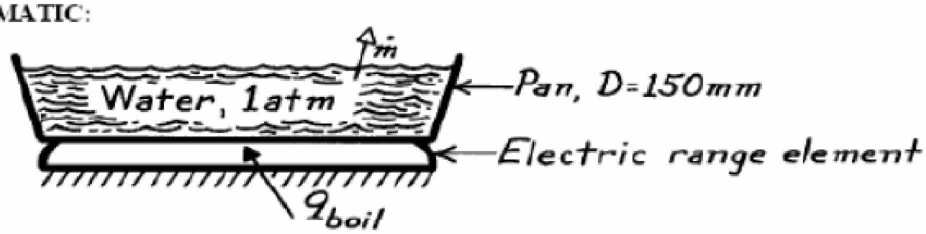
B. What is the rate of evaporation of water from the pan?

C. What fraction of the maximum heat flux for nucleate boiling is required for this surface temperature?

KNOWN: Copper pan, 150 mm diameter and filled with water at 1 atm, is maintained at 115°C.

FIND: Power required to boil water and the evaporation rate; ratio of heat flux to critical heat flux; pan temperature required to achieve critical heat flux.

SCHEMATIC:



ASSUMPTIONS: (1) Nucleate pool boiling, (2) Copper pan is polished surface.

PROPERTIES: *Table A-6*, Water (1 atm): $T_{sat} = 100°C$, $\rho_\ell = 957.9$ kg/m^3, $\rho_v = 0.5955$ kg/m^3, $c_{p,\ell} = 4217$ J/kg·K, $\mu_\ell = 279 \times 10^{-6}$ N·s/m^2, $Pr_\ell = 1.76$, $h_{fg} = 2257$ kJ/kg, $\sigma = 58.9 \times 10^{-3}$ N/m.

ANALYSIS: The power requirement for boiling and the evaporation rate can be expressed as follows,

$$q_{boil} = q''_s \cdot A_s \qquad \dot{m} = q_{boil}/h_{fg}.$$

The heat flux for nucleate pool boiling can be estimated using the Rohsenow correlation.

$$q''_s = \mu_\ell h_{fg}\left[\frac{g(\rho_\ell - \rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,\ell}\Delta T_e}{C_{s,f}\, h_{fg}\, Pr_\ell^n}\right)^3.$$

Selecting $C_{s,f} = 0.0128$ and $n = 1$ from Table 10.1 for the polished copper finish, find

$$q''_s = 279\times10^{-6}\frac{N\cdot s}{m^2}\times 2257\times10^3\frac{J}{kg}\left[\frac{9.8\frac{m}{s^2}(957.9-0.5955)\frac{kg}{m^3}}{58.9\times10^{-3}N/m}\right]^{1/2}\left(\frac{4217\frac{J}{kg\cdot K}\times15°C}{0.0128\times2257\times10^3\frac{J}{kg}\times1.76}\right)^3$$

$$q''_s = 4.839\times10^5\ W/m^2.$$

The power and evaporation rate are

$$q_{boil} = 4.839\times10^5\ W/m^2\times\frac{\pi}{4}(0.150m)^2 = 8.55kW \qquad <$$

$$\dot{m}_{boil} = 8.55kW/2257\times10^3 J/kg = 3.79\times10^{-3}kg/s = 14kg/h. \qquad <$$

The maximum or critical heat flux was found in Example 10.1 as

$$q''_{max} = 1.26MW/m^2.$$

Hence, the ratio of the operating to maximum heat flux is

$$\frac{q''_s}{q''_{max}} = 4.619\times10^5\ W/m^2/1.26MW/m^2 = 0.384. \qquad <$$

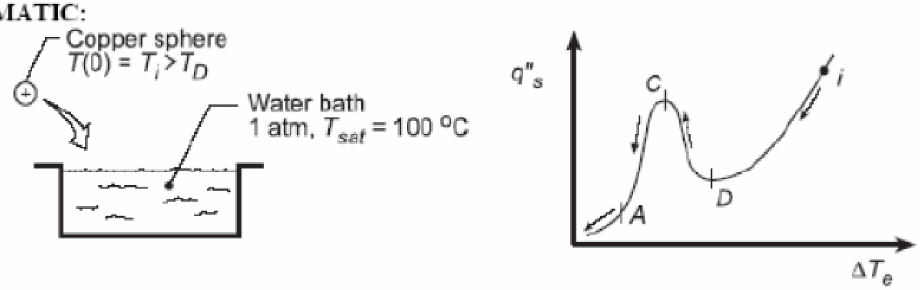
From the boiling curve, Fig. 10.4, $\Delta T_e \approx 30°C$ will provide the maximum heat flux. $<$

7. A small aluminum sphere at a temperature above the Leidenfrost point is dropped into saturated liquid water at 1 atm and 100 °C. Draw the temperature profile that you would expect to observe as the sphere cools from it's initial temperature to 100 °C. (This is a thought problem requiring a qualitative answer, so a rough hand-drawn sketch of the temperature profile is expected, not calculations).

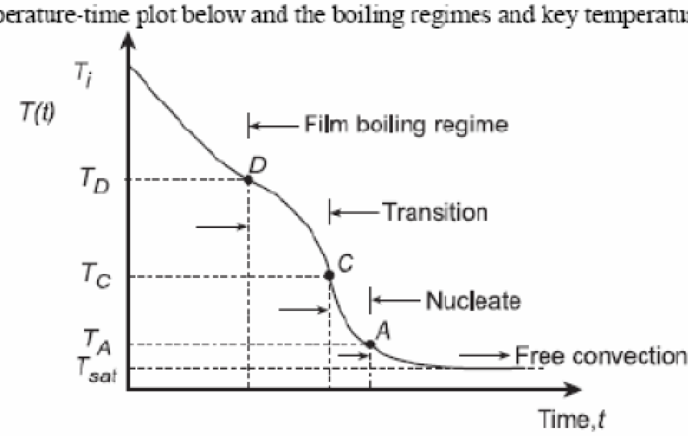
KNOWN: Small copper sphere, initially at a uniform temperature, T_i, greater than that corresponding to the Leidenfrost point, T_D, suddenly immersed in a large fluid bath maintained at T_{sat}.

FIND: (a) Sketch the temperature-time history, T(t), during the quenching process; indicate temperature corresponding to T_i, T_D, and T_{sat}, identify regimes of film, transition and nucleate boiling and the single-phase convection regime; identify key features; and (b) Identify times(s) in this quenching process when you expect the surface temperature of the sphere to deviate most from its center temperature.

SCHEMATIC:



ANALYSIS: (a) In the right-hand schematic above, the quench process is shown on the "boiling curve" similar to Figure 10.4. Beginning at an initial temperature, $T_i > T_D$, the process proceeds as indicated by the arrows: film regime from i to D, transition regime from D to C, nucleate regime from C to A, and single-phase (free convection) from A to the condition when $\Delta T_e = T_s - T_{sat} = 0$. The quench process is shown on the temperature-time plot below and the boiling regimes and key temperatures are labeled.



The highest temperature-time change should occur in the nucleate pool boiling regime, especially near the critical flux condition, T_c. The lowest temperature-time change will occur in the single-phase, free convection regime.