Radiation Exchange between Surfaces: Participating Media

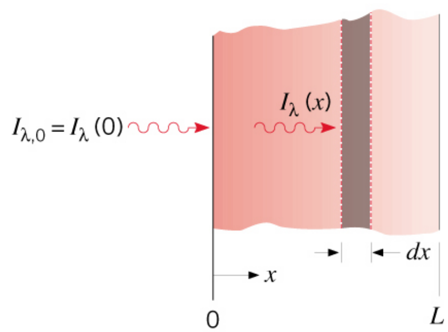
Chapter 13

Section 13.4

General Considerations

- The medium separating surfaces of an enclosure may affect radiation at each surface through its ability to absorb, emit and/or scatter (redirect) radiation.
- Participating media may involve semitransparent solids and liquids, as well as polar gases such as CO_2, $H_2O(v)$, CH_4, and O_3.
- Radiation transport in participating media is a volumetric phenomenon, and for polar gases is confined to discrete wavelength bands.
- Beer's law: A simple relation for predicting the exponential decay of radiation propagating through an absorbing medium.

$$\frac{I_{\lambda,x}}{I_{\lambda,0}} = \exp\left(-\kappa_\lambda x\right)$$



$\kappa_\lambda \rightarrow$ spectral absorption coefficient (1/m)

- Transmissivity and absorptivity of medium of thickness L:

$$\tau_\lambda = \left(I_{\lambda,L} / I_{\lambda,0}\right) = \exp\left(-\kappa_\lambda L\right) \qquad \alpha_\lambda = 1 - \tau_\lambda = 1 - \exp\left(-\kappa_\lambda L\right)$$

- Emissivity of medium. Assuming applicability of Kirchhoff's law:

$$\varepsilon_\lambda = \alpha_\lambda$$

Radiant Heat Flux from an Emitting/Absorbing Gas to an Adjoining Surface

- An approximate procedure involves use of a mean beam length, L_e, to apply emissivity data obtained for a hemispherical gas mass to other gas geometries.

$L_e \rightarrow$ radius of a hemispherical gas mass whose emissivity, ε_g, is equivalent to that for the geometry of interest.

- Emissivity data have been obtained for a hemispherical gas mass with radiating species of H_2O (v) $\rightarrow$ w and/or CO_2 $\rightarrow$ c in a mixture with other nonradiating gases. Results depend on

  T_g – the gas temperature

  p_w, p_c – the partial pressures of H_2O (v) and CO_2

  p – the total pressure of the mixture

  L – the radius of the hemisphere

$$\varepsilon_g = \varepsilon_w + \varepsilon_c - \Delta\varepsilon$$

Fig. 13.19

Figs. 13.17, 13.18

Figs. 13.15, 13.16

$\Delta\varepsilon \rightarrow$ correction factor for mutual absorption of radiation for H_2O (v) and CO_2

- For application to other gas geometries, replace L by L_e.

  $L_e \rightarrow$ Table 13.4

- Rate of heat transfer to a surface of area A_s due to emission from an adjoining gas is

$$q = \varepsilon_g A_s \sigma T_g^4$$

- Net rate of radiation exchange between a black surface and an adjoining gas is

$$q_{net} = A_s \sigma \left(\varepsilon_g T_g^4 - \alpha_g T_s^4 \right)$$

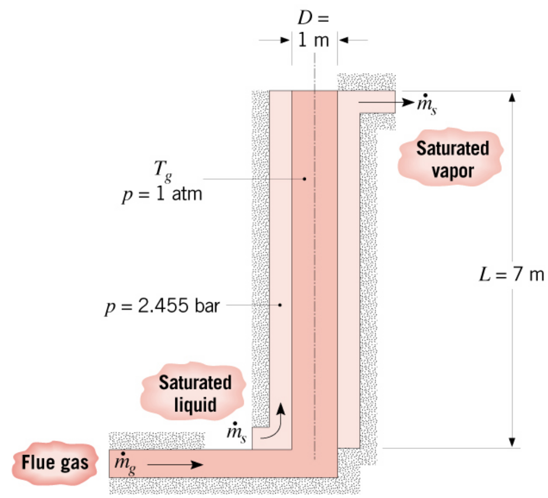
$$\alpha_g = \alpha_w + \alpha_c - \Delta\alpha$$

$\Delta\varepsilon \rightarrow$ Fig. 13.19

Eq. (13.37)

Eq. (13.36)

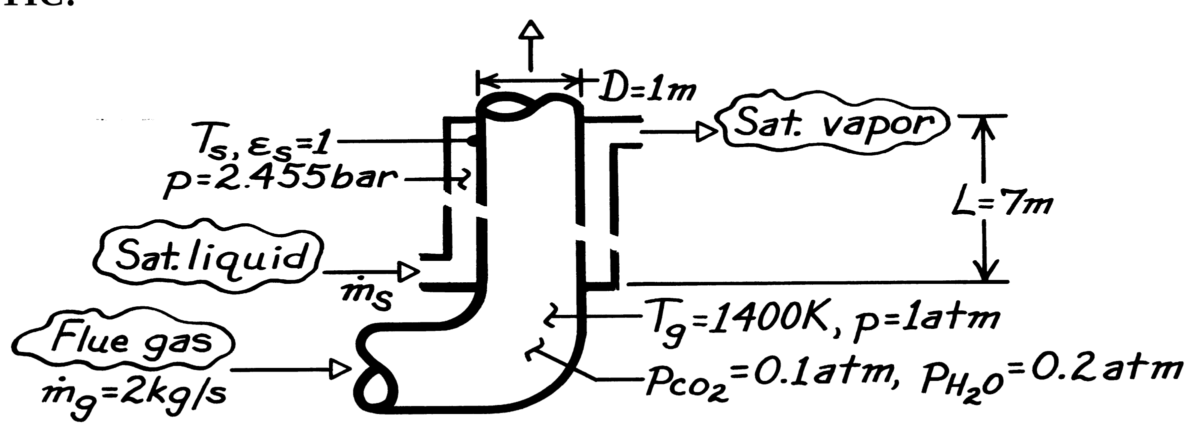
Problem 13.130: Heat recovery from flue gases to convert saturated water to saturated vapor.



KNOWN: Flowrate, composition and temperature of flue gas passing through inner tube of an annular waste heat boiler. Boiler dimensions. Steam pressure.

FIND: Rate at which saturated liquid can be converted to saturated vapor, $\dot{m}_s$.

SCHEMATIC:



ASSUMPTIONS: (1) Inner wall is thin and steam side convection coefficient is very large; hence $T_s = T_{sat}(2.455\text{ bar})$, (2) For calculation of gas radiation, inner tube is assumed infinitely long and gas is approximated as isothermal at T_g.

PROPERTIES: Flue gas (given): $\mu = 530 \times 10^{-7}$ kg/s·m, k = 0.091 W/m·K, Pr = 0.70; *Table A-6*, Saturated water (2.455 bar): $T_s = 400$ K, $h_{fg} = 2183$ kJ/kg.

ANALYSIS: The steam generation rate is

$$\dot{m}_s = q / h_{fg} = \left(q_{conv} + q_{rad}\right) / h_{fg}$$

where

$$q_{rad} = A_s \sigma \left(\varepsilon_g T_g^4 - \alpha_g T_s^4\right)$$

with

$$\varepsilon_g = \varepsilon_w + \varepsilon_c - \Delta\varepsilon \qquad\qquad \alpha_g = \alpha_w + \alpha_c - \Delta\alpha.$$

From Table 13.4, $L_e = 0.95D = 0.95$ m $= 3.117$ ft. Hence

$$p_w L_e = 0.2\text{ atm} \times 3.117\text{ ft} = 0.623\text{ ft} - \text{atm}$$

$$p_c L_e = 0.1\text{ atm} \times 3.117\text{ ft} = 0.312\text{ ft} - \text{atm}.$$

From Fig. 13.15, find $\varepsilon_w \approx 0.13$, and from Fig. 13.17, $\varepsilon_c \approx 0.095$. With $p_w/(p_c + p_w) = 0.67$ and $L_e(p_w + p_c) = 0.935$ ft-atm, from Fig. 13.19 find $\Delta\varepsilon \approx 0.036 \approx \Delta\alpha$. Hence $\varepsilon_g \approx 0.13 + 0.095 - 0.036 = 0.189$.

Also, with $p_wL_e(T_s/T_g)$ = 0.2 atm × 0.95 m(400/1400) = 0.178 ft-atm and T_s = 400 K, Fig. 13.15 yields $\varepsilon_w \approx 0.14$. With $p_cL_e(T_s/T_g)$ = 0.1 atm × 0.95 m(400/1400) = 0.089 ft-atm and T_s = 400 K, Fig. 13.17 yields $\varepsilon_c \approx 0.067$. Hence

$$\alpha_w = \left(T_g / T_s\right)^{0.45} \varepsilon_w \left(T_s,\ p_w L_e T_s / T_g\right) = \left(1400/400\right)^{0.45} 0.14 = 0.246$$

and

$$\alpha_c = \left(T_g / T_s\right)^{0.65} \varepsilon_c \left(T_s, p_c L_e T_s / T_g\right) = \left(1400/400\right)^{0.65} 0.067 = 0.151$$

$$\alpha_g = 0.246 + 0.151 - 0.036 = 0.361.$$

Hence

$$q_{rad} = \pi\left(1\text{ m}\right)7\text{ m} \times 5.67 \times 10^{-8}\text{W/m}^2 \cdot \text{K}^4 \left[0.189\left(1400\text{ K}\right)^4 - 0.361\left(400\text{ K}\right)^4\right]$$

$$q_{rad} = \left(905.3 - 11.5\right)\text{kW} = 893.8\text{ kW}.$$

For convection,

$$q_{conv} = \bar{h}\pi DL\left(T_g - T_s\right)$$

With

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4 \times 2\text{ kg/s}}{\pi \times 1\text{ m} \times 530 \times 10^{-7}\text{kg/s} \cdot \text{m}} = 48{,}047$$

and assuming fully developed turbulent flow throughout the tube, the Dittus-Boelter correlation gives

$$\overline{Nu}_D = 0.023 Re_D^{4/5} Pr^{0.3} = 0.023\left(48{,}047\right)^{4/5}\left(0.70\right)^{0.3} = 115$$

$$\bar{h} = \left(k/D\right)\overline{Nu}_D = \left(0.091\text{ W/m} \cdot \text{K}/1\text{ m}\right)115 = 10.5\text{ W/m}^2 \cdot \text{K}.$$

Hence

$$q_{conv} = 10.5\ \text{W/m}^2 \cdot \text{K}\,\pi(1\ \text{m})\,7\ \text{m}(1400 - 400)\,\text{K} = 230.1\ \text{kW}$$

and the vapor production rate is

$$\dot{m}_s = \frac{q}{h_{fg}} = \frac{(893.8 + 230.1)\,\text{kW}}{2183\ \text{kJ/kg}} = \frac{1123.9\ \text{kW}}{2183\ \text{kJ/kg}}$$

$$\dot{m}_s = 0.515\ \text{kg/s}.$$

COMMENTS: (1) Heat transfer is dominated by radiation, which is typical of high-temperature heat recovery devices having a large gas volume.

(2) A more detailed analysis would account for radiation exchange involving the ends (upstream and downstream) of the inner tube.

(3) Using a representative specific heat of c_p = 1.2 kJ/kg·K, the temperature drop of the gas passing through the tube would be ΔT_g = 1123.9 kW/(2 kg/s × 1.2 kJ/kg·K) = 468 K.