# Free Convection: Mass Transfer

Chapter 9

Section 9.10

# Mass Transfer

- The heat and mass transfer analogy may be invoked if temperature-dependent density variations are negligible relative to density variations associated with species concentration gradients.

In such cases,

$$\overline{Sh}_L \equiv \frac{\overline{h}_m L}{D_{AB}} = f\left(Gr_L, Sc\right)$$

$$Gr_L = \frac{g\left(\rho_s - \rho_\infty\right)L^3}{\rho \nu^2}$$

$$\rho_s = \rho_{s,A} + \rho_{s,B}$$

$$\rho_\infty = \rho_{\infty,A} + \rho_{\infty,B}$$

$$\rho = \left(\rho_s + \rho_\infty\right)/2$$

- Representative Mass Transfer Correlations:

Vertical Plate (analogous to Eq. 9.24):

$$\overline{Sh}_L = C\left(Gr_L Sc\right)^n$$

$$\text{laminar - } C = 0.59, n = 1/4 \quad \left(10^4 < Ra_L < 10^9\right)$$

$$\text{turbulent - } C = 0.10, n = 1/3 \quad \left(10^9 < Ra_L < 10^{13}\right)$$

Horizontal Plate: $\rho_s < \rho_\infty$ at lower surface or $\rho_s > \rho_\infty$ at upper surface.

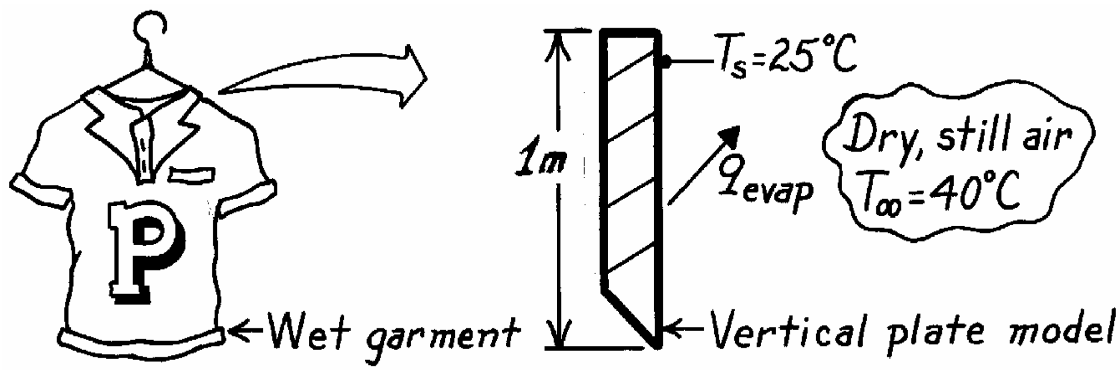
$$\overline{Sh}_L = 0.27\left(Gr_L Sc\right)^{1/4} \quad \left(10^5 < Ra_L < 10^{10}\right)$$

Analogous to Eq. 9.32.

- Consider transport of species A from a horizontal surface facing upward in a quiescent fluid B. If the molecular weight of A is less than that of B, what is the analogous heat transfer problem? If the molecular weight of A exceeds that of B, what is the analogous heat transfer problem?

- If conditions are characterized by both temperature and concentration gradients, how may convection heat and mass transfer coefficients be calculated to a first approximation?

# Problem 9.113: Determination of drying rate per unit width of a garment hanging in dry air.




**KNOWN:** Wet garment at 25°C hanging in a room with still, dry air at 40°C.

**FIND:** Drying rate per unit width of garment.

**ASSUMPTIONS:** (1) Analogy between heat and mass transfer applies, (2) Water vapor at garment surface is saturated at $T_s$, (3) Perfect gas behavior of vapor and air.

**PROPERTIES:** *Table A-4,* Air ($T_f \approx (T_s + T_\infty)/2 = 305K$, 1 atm): $\nu = 16.39 \times 10^{-6}\ m^2/s$; *Table A-6,* Water vapor ($T_s = 298K$, 1 atm): $p_{A,s} = 0.0317$ bar, $\rho_{A,s} = 1/v_f = 0.02660\ kg/m^3$; *Table A-8,* Air-water vapor (305 K): $D_{AB} = 0.27 \times 10^{-4}\ m^2/s$, $Sc = \nu/DAB = 0.607$.

**ANALYSIS:** The drying rate per unit width of the garment is

$$\dot{m}'_A = \overline{h}_m \cdot L\left(\rho_{A,s} - \rho_{A,\infty}\right)$$

where $\overline{h}_m$ is the mass transfer coefficient associated with a vertical surface that models the garment. From the heat and mass transfer analogy, Eq. 9.24 yields

$$\overline{Sh}_L = C\left(Gr_L\, Sc\right)^n$$

where $Gr_L = g\Delta\rho L^3/\rho\nu^2$ and $\Delta\rho = \rho_s - \rho_\infty$. Since the still air is dry, $\rho_\infty = \rho_{B,\infty} = p_{B,\infty}/R_B\, T_\infty$, where $R_B = \Re / \mathcal{M}_B = 8.314 \times 10^{-2}\ m^3\cdot bar/kmol\cdot K/29\ kg/kmol = 0.00287\ m^3\cdot bar/kg\cdot K$. With $p_{B,\infty}$ = 1 atm = 1.0133 bar,

$$\rho_\infty = \frac{1.0133\ \text{bar}}{0.00287\ \text{m}^3\cdot\text{bar/kg}\cdot\text{K}\times 313\ \text{K}} = 1.1280\ \text{kg/m}^3$$

The density of the air/vapor mixture at the surface is $\rho_s = \rho_{A,s} + \rho_{B,s}$. With $p_{B,s} = 1\ atm - p_{A,s}$ = 1.0133 bar – 0.0317 bar = 0.9816 bar,

$$\rho_{B,s} = \frac{p_{B,s}}{R_B\, T_s} = \frac{0.9816\ \text{bar}}{0.00287\text{m}^3\cdot\text{bar/kg}\cdot\text{K}\times 298\ \text{K}} = 1.1477\ \text{kg/m}^3$$

Hence, $\rho_s$ = (0.0266 + 1.1477) kg/m$^3$ = 1.1743 kg/m$^3$ and $\rho = (\rho_s + \rho_\infty)/2$ = 1.512 kg/m$^3$. The Grashof number is then

$$Gr_L = \frac{9.8\ m/s^2 \times (1.1743 - 1.1280)\,kg/m^3\,(1\ m)^3}{1.1512\ kg/m^3 \times \left(16.39 \times 10^{-6} m^2/s\right)^2} = 1.467 \times 10^9$$

and ($Gr_L$ Sc) = $8.905 \times 10^8$. Hence, from Section 9.6.1, C and n are 0.59 and 1 / 4 and the convection coefficient is then

$$\overline{h}_m = \frac{D_{AB}}{L}\overline{Sh}_L = \frac{0.27 \times 10^{-4} m^2/s}{1\ m} \times 0.59\left(8.905 \times 10^8\right)^{1/4} = 0.00275\ m/s$$

The drying rate is then

$$\dot{m}'_A = 2.750 \times 10^{-3} m/s \times 1.0 m\,(0.0226 - 0)\,kg/m^3 = 6.21 \times 10^{-5}\ kg/s \cdot m.$$

**COMMENTS:** Since $\rho_s > \rho_\infty$, the buoyancy driven flow *descends* along the garment.