Gas Drag Simulations of Particles in Protoplanetary Disks

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Abstract

The discontinuity of current solar system formation theories, chiefly concerning time scale, presents an interesting research problem. This paper examines the effect of gas drag on particles in a protoplanetary disk known as the Kuiper Belt. Because significant perturbations were seen in the Kuiper Belt in the short time period that these simulations were performed on 1000 years, this research supports a recently suggested model of a quickly forming solar system.

Introduction

Inside a giant molecular cloud, the early solar system began, giving life to the sun, the earth, to humankind, and to questions about the universe. It is still a mystery how the solar system formed, and there is not currently a working model. There are several models that successfully show the formation of the Jovian planets that do not take into account the smaller, closer planets. Furthermore, there are multiple formation theories that seem to accurately show the formation of the inner planets, but cannot allow for the formation of the outer planets. The ever-present dilemma is the time scale (Tytell 2).

Current research, however, is hopefully approaching a solution to the solar system formation problem. Recently, it has been observed that stars seem to lose their gaseous disks by six million years. This observation creates a problem, as it has long been thought that hot Jupiters require at least ten million years to form. However, it has also been shown that T Tauri stars, thought to have been diskless, appear to have invisible hydrogen disks (Tytell 3). The presence of these hydrogen disks allows for the formation of the inner planets and gives the outer planets enough time to form. In order for this theory to be correct, though, the gas has to decouple from the dust. This research explores the effect of such gas on the dust particles in a protoplanetary environment.

Purpose

This paper analyzes the effect of gas drag on planetesimals in a protoplanetary disk, like the Kuiper Belt would have been to our early solar system. The intention of this research is to contribute to the details of planet formation and to the ideas of planetary astronomy. Further pursuits will include the simulations of inclination in protoplanetary disks.

Background

The Kuiper Belt has also recently become an area of increased interest. The Kuiper Belt is made up of many planetesimals that orbit in this solar system beyond Neptune, between about 30 AU and 100 AU. These planetesimals are believed to be remnants of our solar system formation and are, therefore, extremely important in studies of solar system formation. The simulations in this paper have a disk of planetesimals orbiting a central star of the Sun's mass in an attempt to simulate the conditions of the solar system in protoplanetary form.

Method

We calculated a model of planetesimals by using a fifth order Runge-Kutta algorithm. The Runge-Kutta numerical model was used because it is powerful, easy to program, and abundant, as many types and orders exist (Nazzario). Moreover, Runge-Kutta method uses better approximations by estimating acceleration and velocity at several points in the interval to calculate the next step (Nazzario).

$$y_{i+1} \equiv y_i + \Phi(x_i, y_i, \Delta t)\Delta t$$

Begin with the acceleration:

$$\vec{a} = \ddot{\vec{r}} = \frac{d^2\vec{r}}{dt^2}$$

Split into two differential equations:

$$\vec{v} = \frac{d\vec{x}}{dt} \quad \vec{a} = \frac{d\vec{v}}{dt}$$

Convert to difference equations:

$$\vec{x}_{new} = \vec{v}\Delta t + \vec{x}_{old}$$

$$\vec{v}_{new} = \vec{a}\Delta t + \vec{v}_{old}$$

Fifth Order Method

This fifth order method is based on Butcher's scheme, using six evaluation points and a constant time step. Many versions of this code have been used over the past few years to do simulations of this kind. Equations for new position and velocity are the following:

$$v_{i+1} = v_i + \frac{\Delta t}{90}\left[7M_0 + 32M_2 + 12M_3 + 32M_4 + 7M_5\right]$$

$$y_{i+1} = y_i + \frac{\Delta t}{90}\left[90v_i + 7hM_0 + 24hM_2 + 6hM_3 + 8hM_4\right]$$

(Nazzario)

There was a gas drag algorithm added to this Runge-Kutta program. The Epstein Law for drag force was used, utilizing mean free path equations. The Epstein Law states that F=3/4 p s^2 v **v** (Levy 1045). The code can be improved by using a variable time step for the density in the gas drag algorithms. This would cause the density of the gas to increase over time, which better simulates actual conditions. Further plans include adding an inclination algorithm to the code.

Initial Conditions

The planetesimals disk orbits a star of one solar mass. All runs had 440 planetesimals that consist of eleven rings with 400 parts per ring, each ring having a density of 3 g/m^3, two protoplanets, and a time scale of 1000 years. Four runs were performed with no gas drag for comparison. The experimental value in each run was the distance of the disk from the central star and thickness of the disk. Run one was performed with the disk at a distance of 10 AU from the sun and reaching a distance of 20 AU. One protoplanet was placed at the inner edge of the ring and the second at the outer edge. Run two was from 6 AU to 10 AU, run three from 10 to 15 AU, and run four from 15 to 20 AU. The gas drag runs took their initial conditions from each of the non-gas drag runs, as well as varying the density in the second two runs. The density used for the first two runs was 3.256 x 10^-19 kg/m^3. The density for the second two runs was reduced by a factor of ten. The temperature used for all four runs was 500 Kelvin, varied/constant by distance.

Results

Graphs and Tables

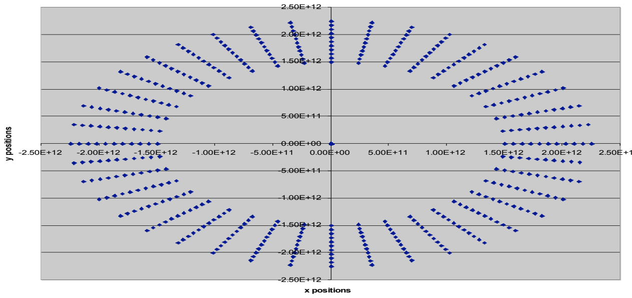

Figure 1: Graph of x vs. y positions of initial conditions

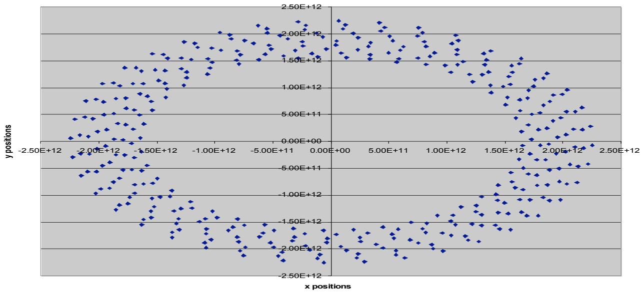

Figure 2: Graph of x vs. y positions for first non-gas drag run (10-20 AU)

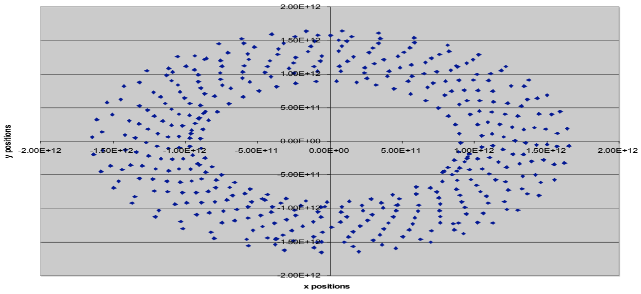

Figure 3: Graph of x vs. y positions for second non-gas drag run (6-10 AU)

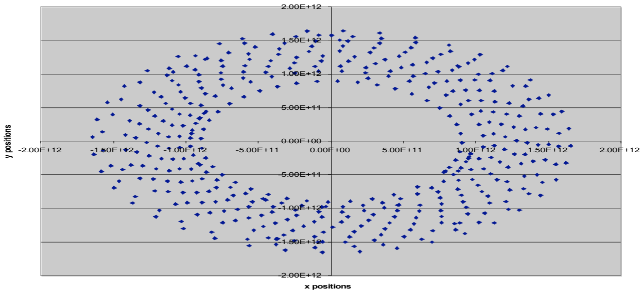

Figure 4: Graph of x vs. y positions for fourth non-gas drag run (15-20 AU)

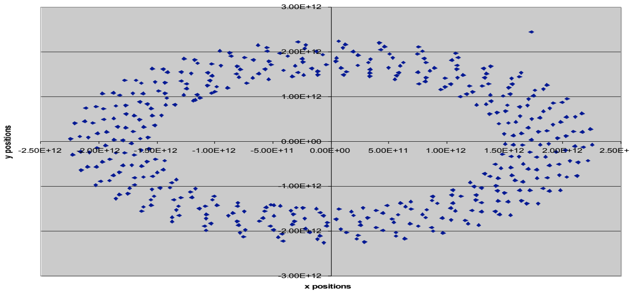

Figure 5: Graph of x vs. y positions for third non-gas drag run (10-15 AU)

The density seemed to be too high in the first two gas drag simulations. All the particles were ejected out of the system. Yet, the ensuing runs with the density of the disk decreased were successful.

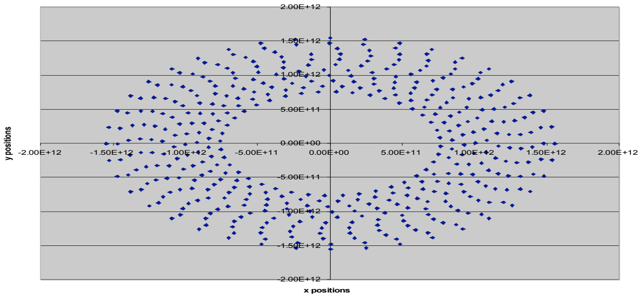

Figure 6: Graph of x vs. y positions of the first successful gas drag run (10-20 AU)

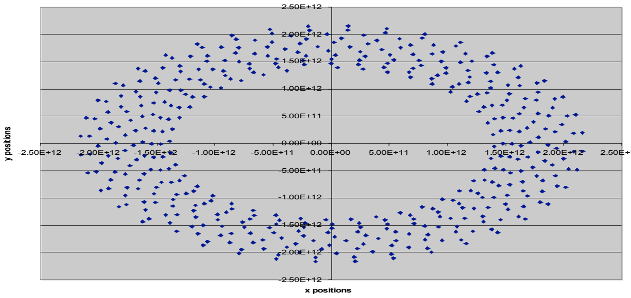

Figure 7: Graph of x vs. y positions of the second successful gas drag run (6-10 AU)

Conclusion

The run performed on a disk of ten AU seems to have been unperturbed, yet the run performed from six to ten AU seems to have some clumping, which was most likely caused by the gas drag. It appears that the gas drag equalizes the disk when comparing the position graph of the gas drag run from six to ten AU to the corresponding non-gas drag graph. The non-gas drag run had more clumping and a more uneven distribution of particles.

The observation of events occurring in this small time period of only 1000 years corroborates the theory of a quickly forming solar system. Subsequently, further research with different gas drag parameters and inclination of orbits in the disks will be investigated.

Acknowledgments

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Christy Hyde is a senior from Huntsville, AL. She currently serves as the Sigma Pi Sigma Physics Honor Society President at UA and is a Society of Physics Student Associate Zone Councilor. In addition to pursuing a major in physics and a minor in astronomy, Christy has conducted extensive research at both Baylor University and the Los Alamos National Laboratory through the National Science Foundation's REU Program.