Some Thought on Infinite Divisions and the Concept of Infinitesimals in Relation to Time: A Theory on the "Instant"

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Introduction

I will argue that time is not physically infinitely divisible--that is, there is a unit of time that is physically indivisible. I will use the concept of an instant. An instant is a unit of time such that an object exists for a single instant only if it neither perdures nor endures. That is, if an instantaneous object exists at instant *t*, there is no time before or after *t* at which the object exists. Because change involves having different properties at different times, the instantaneous object cannot undergo change: it exists at but a single time.

I assume that the instant is the basic unit of time--that one cannot divide an instant into two separate times, and I argue that there are a finite number of instants within any duration of time. It is not physically possible for an object to exist for less than an instant and not for an entire instant, nor for an event to occur in less than an instant but not last for an entire instant. This has some strong implications, saying that a measurable time unit, such as a second, is not infinitely divisible. Of course, by claiming that, I owe an answer to, "How much time is an instant?"

To answer this, I introduce Planck time and argue that the Planck time unit *is* physically indivisible. I will also argue that a mark in favor of my concept of a physically indivisible unit of time is that it preserves many commonsense beliefs about the nature of time. One is not forced to side with peculiar or extreme positions to accept my argument.

Time and Infinite Divisions

The notion of infinite divisibility and its implications are central to formulating my view. To introduce infinite divisibility, I use the concept of limits, which is best explained by way of the following example derived from Zeno:

Suppose you were told to cross a room. First, you must reach the point 1/2-way to the wall. Then, you must reach the point 1/4-way to the wall. Next, you must reach the point 1/8-way to the wall, etc. You have an infinite number of tasks ahead of you before you can cross the room, thus you need an infinite amount of time to cross the room.

In so far as no one can complete an infinite number of tasks in a finite amount of time, and because any normal walker can cross a room in a finite amount of time, there must be something wrong with the above view.

This would not work in practice, because at some point a person's motor skills would not allow him to move his foot half of the distance to the wall, even if he could determine the point that is half the distance to the wall. There is a point at which human ability to move a distance smaller than the previous step would fail. This would work the same for any object put to this test of infinitely decreasing distances, be it machine or human. There is a point at which "moving in an increment half the size of the previous increment" is literally not physically possible in the world in which we live.

The flaw in the above Zeno paradox is that it assumes that points are not extended. To pass through any amount of space will require passing through an infinite

number of points, which will require an infinite number of moments, thus an infinite amount of time. With the above justification, I assume that there is a minimal length; that is, points are extended in space.

Some philosophers might exclaim, "Hold on! You're saying that it's not physically possible in the world in which we live that someone (or something, they might say) could move in infinitely decreasing step sizes. However, it is logically possible, so how can you refute it?" To this philosopher I reply that just because it is possible *logically* that I run a mile in 3 minutes doesn't mean that it is *physically* possible. There are limitations in the physical world that keep certain "logical possibilities" from being actual in the world in which we live. To make my distinction between logical and physical possibility, I offer the following definitions:

- x is logically divisible =Df x is strictly made up of discrete proper parts y and z; and it is logically possible that y exists and z exists, but x does not exists.
- x is physically divisible =Df x is strictly made up of y and z; it is physically possible that y exists, z exists, and there is nothing strictly made up of y and z.

There is a length, *L,* that is logically divisible (it is logically possible that either *m* or *n,* which are discrete parts of *L,* exist but *L* does not), but that is physically indivisible (*m* nor *n* can exist if *L* does not exist; it is not physically possible that *m* and *n* exist and that there is nothing strictly made up of *m* and *n*--namely, *L*)

We now have reason to believe that there is a minimal length (the exact figure for this length is discussed later). To introduce the minimal duration, I will use another paradox by Zeno concerning Achilles:

Achilles decides to race a tortoise, giving the tortoise a head start at point A. To pass the tortoise, Achilles must reach point A where the tortoise originally started. By that time, however, the tortoise has already moved ahead to point B, so Achilles must then reach B. When Achilles reaches B, the tortoise has already moved ahead to point C, etc. Achilles will never pass the tortoise.

Obviously, the above example does not hold true in the physical world. Anyone can give a tortoise a head start in a race and still win. While the above example does well by assuming that points are extended in space, the flaw is that it assumes that instants are not extended in time. If instants are not extended, then time can be divided infinitely. And if points *are* extended in space, the amount of time required to pass through a point contains an infinite number of instants. Therefore, you need an infinite number of instants to pass through any distance in space. Following this, I assume that instants are extended in space.

To strengthen this assumption, consider that *an instant is durationless, and, let's assume, that one second is made up of an infinite number of instants? Since the duration of an instant is d=0, how does a sum of these instants ever equal the duration of a second? Surely the smallest time unit must have duration, or else all other time units—which would be made up of the smallest units—would never add up to more than* ∞ *x (d=0) = 0. Just as there must be a minimal length, this problem is solved if we have a minimal duration.*

Conventionally, "instant" is used to refer to a durationless time. On my view, however, an instant is a duration of a non-arbitrary minimal length. An instant is the basic unit of time and all measurements of time are made up of instants. It is not such that an instant is durationless, but it is true that the instant is the minimal duration.

Any extended unit of time (e.g. one second) can be divided by 10 and made into tenths of a second. These tenths can then be divided and so forth until you reach femtoseconds or whatever extended unit you want to reach. At what point does one reach "instants"? Are there infinitely many instants in a second? Perhaps in thought, but surely within the practices of the world we live you cannot just keep dividing time forever, just like we cannot keep dividing the size of our footsteps forever. This would imply that there is an infinite amount of instants (and instantaneous events) in one second and also an infinite number of instants (and instantaneous events) in a half-second or femtosecond. It seems sensible to think that if there are an x number of instantaneous events in half a second, there should be *2x* number of instantaneous events in a whole second. If time is infinitely divisible, this is not true. There is the same amount of events in any unit of time, namely an infinite amount. This seems counterintuitive.

We can, in thought, think about dividing a second into a million-billion-trillion other increments of time. But, what purpose does this serve? Thinking about unicorns does not make them exist, just as thinking about me running a mile in three minutes does not give me that ability, just as thinking about a million-billion-trillionth of a second does not mean it exists. Once again, a mistake is made into thinking that just because there is an infinite number of points between two things logically means that there are an infinite number of points between two things physically. There are not an infinite number of tasks between you and your goal of walking across the room. You can describe these tasks in an infinite amount of numbers in thought, but you cannot divide them infinitely in practice. You can describe an interval of time as an infinite amount of numbers by simply taking an amount of time and dividing it by 2 (or whatever number you want to use) over and over gain, but time is not physically infinitely divisible. So, all the worse for a million-billion-trillionth of a second. There must be a point at which the divisions of a second that we can conjure up *in thought* cease to be what exist in the physical world in which we live. Therefore, the answer to, "What is the limit of the divisions a second can undertake?" seems to rest on the answer to, "What is the duration of an instant?"

Interesting Implications for Time Theorists

Ultimately, the size of the "present" is dependent on these answers. Trenton Merricks and other elastic presentists argue that months, years, etc. can be the present. Trenton Merricks calls this supposition "odd" but defensible: "Suppose events are property exemplifications (such as *O*'s being red). Why couldn't *O's being red* be wholly present at more than one time?" (Merricks 523-31). Merricks would suggest that the 21st century is a time that *did* exist and is also a time that *presently* exists, thus the "present" is elastic and can be defined in many ways. Strictly, however, the twenty-first century is not a time that both did exist and presently exists. In presentist terms, the twenty-first century both did exist and presently exists only in the sense that there did exist a time that existed after 12:00 AM on January 1, 2001 but before 12:00 AM on January 1, 2101, and there presently exists a time that exists after 12:00 AM on January 1, 2001 but before 12:00 AM on January 1, 2101 (Hestevold 491-510). Thus, I will not refute Merricks, but I will say that his definition of the present is skewed from the typical definition of the present being *now*. I believe that the present (in the traditional sense), if it exists, is exactly an instant long. My theory does not hinge on Merricks being wrong, so I will not discuss the possibility of an extended present any further.

I do refute the common view that the present is infinitely thin. As said before, the present is a duration. If there is no unit of time smaller than an instant that exists apart from an instant, then the present cannot exist for a duration less than the duration an instant since there couldn't be any event to occupy a present that is shorter than an instant. Therefore, the present must be *at least* as long as the duration of an instant.

A major flaw in transient time has been the lack of a definable present. Questions about the thickness of the present (What would it mean to have an infinitely small "now"? What if it is very large? What about a variable size of the "now," like Merricks suggests?) are often the basis for objections to transient time. My account for the size of the present offers the transient time theorist a reply to this objection, thus strengthening any case for transient time. And, since presentism relies on transient time being correct (Hestevold 491-510), my account for the size of the present also strengthens the case for presentism being correct.

Although I have referred often to the "present," I do not assume that presentism, transient time, nor static time is true. My argument of a non-infinitely small "present" can be taken to mean: 1) that the actual present has a duration--for the presentists and transient time theorists, or 2) that the framework of 4-D spacetime is made of small durations, and that a specific state of the universe takes up a non-infinitely small duration in 4-D spacetime--for the static time theorists. I do believe, however, that one should not reject transient time on grounds that it cannot offer a coherent account of the duration of the present, because I do offer a clear account for the duration of the present.

I now will discuss the length of a duration of an instant.

Planck Time and the Instant

The Planck length is developed using an equation based on gravity, the speed of light, and a constant developed by physicist Max Planck. The Planck length is accepted by most in the field of modern physics as the smallest spatial dimension that exists. The Planck length would be the dimension of a "blockule," mentioned later. It is roughly equal to 1.6×10^{-35} m or about 10^{-20} times the size of a proton.

The Planck time is the time it would take a photon traveling at the speed of light to cross a distance equal to the Planck length. The standard number used for Planck time is 10^{-43} seconds. (Equations for the Planck units are given at the end of the paper, but any further analysis of Planck units is excluded here to avoid a detour through pages of physics.)

Planck time is the smallest unit of time that physically exists apart from another whole unit. That is, there does exist one-half Planck time units, but they exist only in that they are part of whole Planck time units. Units of time smaller than Planck time units are meaningless when taken independently of whole Planck time units. My brief justification for the significance of the Planck time is that is not possible for any object to travel faster than light.

The actual smooth "arrow" of time that we typically think exists is actually constructed of many Plank times, and it is not physically possible for a time interval smaller than this to be accounted for, measured, or detected. Since no non-overlapping task can happen faster than a photon of light can travel across a Planck length, there are no events that independently occur in less than a Planck time unit. Thus, any smaller

time unit is incapable of measuring any event, and no such unit is meaningful in the physical world.

The following analogy may make the Planck units easier to understand. In a photograph, the picture looks very clear. Every detail in the picture looks very smooth and there does not appear to be any roughness to the photograph. However, if one magnifies the picture enough, one can see that the photograph is actually very grainy--there are individual pixels of color that are not apparent in the unmagnified picture. The actual world is exactly the same way. We all see things move smoothly across space, but if we could magnify things down to the sub-atomic scale we would see that objects appear at one point then appear at another point without existing at any point in between--an object would be in one pixel of the picture then appear in the next without "flowing" across the pixel. Movement is "grainy" on the very small scale, but we do not detect this because we cannot detect such small spatial distances nor can we detect such small time increments, just as we cannot detect the individual pixels in a photograph without magnification.

One physicist one explains this in terms of blockules:

As you go further and further down in size, we should wonder if we will eventually reach a solid granularity in our Universe. IE, the Universe might be divided into blockules. A quantum particle can be in one blockule or another, but not in between, nor in both. It would be like pixels on a computer screen. Of course, detecting blockules would normally be impossible, because we are all moving with respect to the fabric of space-time. (This information was found on a quantum physics forum at www.everything2.com. As of November 5, 2003 the server was being physically moved, so the exact link to the forum is no longer available.)

The question of "What is an instant?" seems now to be more answerable. The instant, the basic unit of time, is the Planck time, so talk of time units is reducible to talk of Planck times. It now follows that any talk of instantaneous things is talk of "things that are in existence for exactly 10^{-43} seconds."

Some Problems Addressed

Here I offer replies to some objections to my view, and I deal with a few formalities that have arisen throughout my argument.

One might object that my concept of an instant itself implies that there are times shorter than an instant: "Wait! What if something exists for a shorter time than 10^{-43} seconds? If x exists 10^{-43} seconds then it exists for 10^{-86} seconds. This thing exists, and it is unreasonable to say it exists for less than an instant, since the very meaning of *instant* implies that you cannot divide an instant." To this objector, I offer this analogy: Thinking in terms of "time flow" and "infinitely small increments of time" is like talking about analog time. In analog time, time units are infinitely divisible. A moving object flows from one point to the next—if a moving object exists at point a then later at point b, there are infinitely many points between point a and point b at which the object also exists.

However, time can only be applied as analog on the macroscale, the scale at which our naked eye can detect things. When someone wants to measure time on the microscale (or in my case, when someone wants to philosophically point out what the microscale unit is), the scale that magnifies things to the subatomic level and detects time

increments as small as Planck times, he discovers that time is digital. A thing either completely exists in a Planck time or it does not exist in that time at all. There are no things that exist for, say, exactly 1_ Planck times. Once a thing is in existence for longer than 10^{-43} seconds, it is automatically in existence for two [or more] Plank times. An object warps between two adjacent Planck times with no "half-way crossing over" or anything of that such; in one instant a thing is in one Planck time, *P*, and the very next instant it appears in the adjacent Planck time, *P'*. (Whether the adjacent Plank time must lie towards the "future" is irrelevant to the point being made and is the matter of determining the nature of Time as a whole).

Another objector may ask, "What is the nature of the connection between these individual time units?" There is not a need to postulate extra "stuff" that is between each unit on the plane or line or time, nor is there something new we have to postulate to explain how the Planck times are connected. The time units exactly follow each other. Like the pixels of the photograph, there is not anything in between each block that keeps them apart. The time units are separated by the discrete events that happen in them. Planck time 42 becomes Planck time 43 at 10^{-43} seconds after Planck time 42; they are distinguished by the fact that the universe is in one state with certain properties at Planck time 42, and it is in a different state with different properties 10^{-43} seconds later at Planck time 43. The photograph pixels are separated by the division where the arrangement of color in one pixel becomes different from the arrangement of color in the adjacent pixel.

The connection between each time can be whatever connection one would postulate to describe the connection between two different times in the classical infinitely divisible time view. Causation, for example, is often used to describe how today is connected with yesterday, or how 5:00 is connected with 5:01. Two infinitely divisible times are connected by the single-direction arrow of causation: *t1* causes *t2* and that is why *t2* is after *t1* and not vice versa. Causation can also be used to connect Planck times: t_P2 is directly after t_P1 because the state of the universe at t_P1 caused the state of the universe to be like it is 10^{-43} seconds later at t_P2.

There still may be a critical philosopher to make the following objection: "If there is an event, E, that occurs in a single Planck time, there is a) the event of E beginning and b) the event of E ending--so events can occur that do not last an entire Planck time." However, there cannot be an event that lasts for, say, one-half of a Planck time because things either occur in Planck times or they do not. One cannot look at the grainy checkerboard of time and say, "This event begins at the beginning of Planck time 236 and ends at the end of Planck time 236," and still get by with implying that there are two discrete events occupying half of a Planck time. Whoever said that events must have a beginning and an end never said the beginning and end cannot be at the same time or even be the same event.

Another possible objection follows: "There is the event, *n*, of E occurring over the first half of Planck time 236 and the separate event, *m*, of E occurring over the last half of Planck time 236. How can you say that these events do not last for less than a Planck time?" An event requires a duration of time to occupy, and there is no duration of time smaller than a Planck time for an event to occupy under the laws of physics in this world. All that can be said is that E occurs in Planck time 236. No event can occur in Planck time 236 unless it occurs for all of Planck time 236. The event of E occurring over the first half of Planck time 236 and the event of E occurring over the last half of

Planck time 236 does not pose a problem. The event *n* did not magically find a way to cause *m* within a unit of time smaller than a Planck time. The cause of *n* is whatever sparked E to occur in a time before 236. The cause of *m* is that same event, and the cause of *m* occurring only in 236 and not in 237 is the fact that the universe is in a different state at 237 such that E is no longer a part of the universe, thus *m* cannot be part of the universe since it was part of E. Both events, *n* and *m*, occur in Planck time 236, as do the other many separate events that one can isolate at that time, such as the Earth being covered in water *and* Mars being covered in red dust. The objector could go on and on (there is the event of the event of E occurring over the first half of Planck time 236 occurring over the first quarter of Planck time 236 and over the second quarter) and would arrive only at an infinite number of ways to describe a single event. E occurs at Planck time 236; it does not occur over the first half of the time unit then occur over the second half. The property of "existing in Planck time 236" is either had by an event or it is not.

This serves to change the model of time that is most often used. On the macroscale, time appears to lie along a smooth plane or line, but as the most micro of time scales are analyzed, time becomes a grainy plane or line, made much like a checkerboard where each square is a Plank time and events either fully exist in that time or they do not exist in that time at all. The Planck time, with respect to that event, is either on or off--no "in betweens" are possible. Motion, therefore, is not smooth and analog on the smallest scale. An atom, *a*, spinning wildly away from a molecule exists at point *k* at Plank time *P*, then at the adjacent Planck time, *P'* ($P + 10^{43}$), *a* exists at point *k'*. Whatever these two points may be, and whatever the interval between them (*k'* - *k*) might be, there is no point between *k* and *k'* at which the atom *a* exists. It exists at one instant at point *k* and the very, exact next point it exists at is point *k'*. The object appears at *k* then reappears at point *k'*.

Some wonder if teleportation is possible, and to them I offer the above theory as evidence that it occurs every "instant." (Teleportation is used in the standard science-fiction sense to mean that some physical object is at one point at one time, and then it appears at another point at another time without appearing at any place or time in between.)

Loose Ends Tied

I do not address the possibility of modal realism. For one reason, such a discussion would carry attention away from the topic I wish to undertake. For another, the actuality of modal worlds has little bearing on my thesis. I will make a disclaimer that if modal worlds *do* exist, then any statements I have made pertain to the physical world that I live in and not necessarily to any other world. A world with different laws of physics (i.e. a world with something that travels faster than the speed of light, perhaps "instantaneously" in the conventional sense) may well not follow the demands of my thesis. However, whether or not modal realism is correct, my thesis still applies to the physical world that I live in.

I discuss the discretion of time using the photograph pixels analogy and the blockule hypothesis. Doing so, I assume that time is discrete. However, it may be possible that time is continuous *and* Planck time is still the minimal duration. Transient time implies continuity and forces there to be a "present." In a transient time + continuity

case, the Planck time would be the size of the present. Nothing can happen in a time shorter than the Planck time, so the present has to be a duration equal to the Planck time. I do not find this easy to imagine, but I do not have a reason to believe that it is implausible. I think that time *is* discrete, thus I spent some length discussing discrete time. However, my argument against continuity goes no further than the implied reasons in my previous discussion. Regardless, my thesis does not rely directly on discretion or continuity.

Appendix: The Physics of Planck Time

My explanation of the physics of Planck time was brief and nontechnical to avoid blurring the metaphysics of time with the physics of time. I offer here, however, a sketch of the physics of Planck time.

The Planck constant comes from the quantum physics equation $pq-qp = -i_$, and is noted as constant _ (h-bar). All Planck units rely on physicist Max Planck's constant, which is the fundamental constant of nature that arises mostly in quantum mechanical problems. The constant sets the scale at which the uncertainty principle of quantum theory becomes important, and it quantifies our inability to simultaneously know everything one might choose to know about the world and any given time. Using the Planck constant, we can quantify the minimal length and time (and other measurements not addressed here, including basic ones, such as mass, and non-basic ones, such as radii and wavelengths) at which special relativity and quantum gravity begin to take effect. In simple terms, gravity and other physical forces have no effect on a particle until unit sizes reach the Planck scale. All particles have a force and mass; thus, the smallest units in the universe are Planck units.

The Planck unit for length is given by the following equation:

$$\lambda_P \equiv \sqrt{\frac{G\hbar}{c^3}} = 1.6161 \times 10^{-33}\ \mathrm{cm} = 1.6161 \times 10^{-35}\ \mathrm{m}.$$

where λ_P is the Planck length. This is the quantum unit of length, and would be the width of the "grain" that one would see on the most micro of length scales. Nothing can be smaller than this unit of length without supposing the possibility that the laws of physics are different in another world.

The equation for Planck time is $t_P \equiv \sqrt{\frac{G\hbar}{c^5}} = 1.35125 \times 10^{-43}\ \mathrm{s},$

where t_P is the Planck unit of time. This is the time it takes for one photon of light to travel across the Planck length. Planck's grounds for saying that such a "smallest unit" exists lie in Einstein's theory that nothing can travel faster than light. Supposing for philosophical argument that something can travel faster than light is supposing an alternate world where the laws of physics are different, and thus my argument does not apply to any such case (nor do Planck units apply in such a case).

For the above equations, the following values are used:

G is the universal gravitational constant,
6.673×10^{-11} N m^2 kg^{-2}

h is Planck's constant, 6.6262×10^{-34} J s

h-bar is Planck's constant reduced for consideration of angular momentum, *h-bar=h/2,* 1.0546 x 10^{-34}

c is the speed of light, 2.99792458 x 10^{8} m s^{-1}

Reference for All Constants and Equations

Shu, F. H. *The Physical Universe: An Introduction to Astronomy.* Mill Valley, CA: University Science Books, p. 397, 1982.

References

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